Phase transitions under constraints: from confinement to complex networks





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"Complexity, Criticality & Computation" (C3-2017)

THE UNIVERSITY OF

Sydney, 11-13 December 2017



https://www.theguardian.com/cities/2014/feb/18/slime-mould-rail-road-transport-routes

Porous rocks





Grain Pore

http://perminc.com

Membranes in biology



Membranes in chemistry

Gas separation and filtration

• Fuel cells





Scientific Reports 6, 20430 (2016)

https://physics.nist.gov/MajResFac/NIF/pemFuelCells.html

Phase transitions under constraints: from confinement to complex networks

Phase transitions of confined fluids

• Fluid transport in channels

Nucleation



Wikipedia: nucleation



Journal of Thermal Science (2012), 21

Modelling the interface

Current methodology

density profile is a result of free energy minimization, assuming smooth variables

> surface tension is derived from density variation



Challenge

apply methodology of planar interface to spherical interface

problem of singularity
varying surface area



Density profile



Phase transitions under constraints

Problem

small bubble is not stable?



Solution

small bubble is not stable!



- Closed system
 - fixed amount of molecules, which can be in either of two phases: gas or liquid
- Commpressible fluid
 - density may be adjusted

Capillary model



Thermodynamics: $T dS = dU + p dV - \mu dN$

Equilibrium:

$$\mu_{in}(p_{in}, T) = \mu_{out}(p_{out}, T)$$

$$p_{in} - p_{out} = \frac{2\sigma}{R}$$

Capillary model



Phase transitions under constraints

K. Glavatskiy

Phase diagram





resulting fluid density is lower than the coexistence one - «metastable» region



• Confinement:

- energy redistribution under constraints
- Altered phase diagram
 - closed system + compressible fluid
- Effect is larger for smaller system

- Apparent negative compressibility
- Restrictions on cluster size

Porous transport





Desalination 336, 97-109, (2014)

Carbon nanotube membrane

Membrane model

• Energy profile at CNT entrance



Viscosity

Gravelle et al, J Chem Phys (2014)

Walther et al, Nano Lett (2013)





Pore entry

$$\Delta_s P = Q C \eta \left(\frac{1}{r_a^3} - \frac{1}{r_b^3} \right)$$

Poiseuille

$$\frac{\Delta_P P}{L} = Q \frac{8\eta}{\pi r^4 + 4\pi r^3 s}$$

Bernoully

$$\Delta_B P = \frac{Q^2}{2\pi\rho_b} \left(\frac{1}{r_a^2} - \frac{1}{r_b^2}\right)$$

- ΔP pressure drop
 - Q flow rate
 - r pore radius
 - L pore length

Pore blockage



0

Particle 4

Coordinate

Fluid structure



density inside the pore

$$\rho(x) \Rightarrow \rho_a$$

density across the interface

$$\rho(x, z) \Rightarrow \rho(z)$$

Flow into CNT



Hydrodynamic resistance:

bending of the flow lamina at a geometrical obstacle



Thermodynamic resistance:

phase difference between inside and outside the membrane



Gibbs surface





• Excess resistance

$$R_{s} = \int_{z_{-}}^{z_{+}} r(z) dz - r_{-}(z_{s} - z_{-}) - r_{+}(z_{+} - z_{s})$$

Adsorption isotherms: temperature



Adsorption isotherms: temperature



Adsorption isotherm: pore size



Adsorption isotherm: pore size



Comparison with internal resistance

• 5 Å

• 13 Å



Length of the nanotube, which has the same internal resistance as the interfacial resistance

Resistance vs pore size



Resistance vs pressure



• Confinement:

- energy redistribution under constraints
- Altered phase diagram
 - closed system + compressible fluid
- Effect is larger for smaller system

- Phase transition leads to extra resistance
- Interactions with network are relevant

Acknowledgments

★ Thanks to

- David Reguera
- Dick Bedeaux
- Suresh Bhatia
- Peter Daivis



- ♦ J. Chemical Physics 138, 204708 (2013)
- ◆ Langmuir 32, 3400 (2016)
- ◆ J. Membrane Science 524, 738-745 (2017)