>> Welcome to the podcast series of Raising the Bar Sydney, raising the bar in 2019. So, of 21 University of Sydney academics, take their research out of the lecture theatre, and into bars across Sydney. All on one night. In this podcast, you'll hear Anna Romanov's talk, hidden symmetries. Enjoy the talk.

[Applause]

>> Hello? My name is Anna, and I am a mathematician. And it still feels a little bit weird for me to say this to you. Because if you'd met me as a little girl, and asked me what I wanted to be when I grew up, there's absolutely no way I would have told you that I wanted to be a mathematician. I might have told you I wanted to be a veterinarian, maybe a marine biologist, possibly even a professional soccer player, if I was feeling feisty that day. But, a mathematician? I mean, is that even a job? Well, yes, it is a job. It's my job now. And as an adult with the benefit of hindsight, I realise that little girl Anna, actually had all the makings of being a mathematician. She just didn't know it. So, I grew up in the western United States, in a state called Nevada. And every November, my family would take a road trip to Southern California, to spend the big American holiday of thanksgiving with my cousins. We would leave for my hometown in Carson City, Nevada, early in the morning and drive all the way to Fresno, California. This drive would take us directly through California Central Valley, which is the main agricultural region of the state. If you're familiar at all with California's geography, you might know that this part of California is covered with almond orchards. Eighty percent of the world's almond crops are grown in California Central Valley. So, on these drives, I'd be sitting there in the back of my parent's car, bundled up in blankets against the November chill, staring out the window, at rows and rows of almond trees. And if you've had the experience of driving through farmland, you know what this looks like. You know what it looks like to stare directly at a car window, at crops that are planted in a row. You see this infinite sea of lines, heading off into the distance. As I looked out the window, and looked at these lines, zip-zip-zipping past me, I would imagine that they weren't actually parallel lines. But that instead, there was some point very far away, where they all met. And that what I was seeing, were actually the very ends of some vast collection of rays, that were radiating out from this central point, like spokes on a bicycle. In my little girl imagination, these rays were the legs of a giant spider, that was rolling along the side of the car and keeping us company as we drove. What I was actually doing as a child, is I was seeing patterns in the world around me, and I was expecting them to be part of some bigger hidden symmetry, that I couldn't see with my own eyes. And what's funny, is that this is exactly what I do now; as a professional mathematician. My research is in a type of math called representation theory, which builds a mathematical language as symmetry. I no longer search for patterns in the physical world, like I did as a child. But now I search for hidden symmetry in the mathematical world. And when I find it, I work to understand what implications the symmetry has, on the world that contains it. So, what I want to do today, is I want to tell you two stories of hidden symmetry. At the centre of each of these stories, is a beautiful idea,

which manifests itself as a mathematical theorem. But it doesn't make sense to talk about ideas in isolation. Ideas don't exist in a vacuum. Ideas come from people. So, to tell you these two stories, I have to tell you about the people who had these ideas. And what I love most about these stories, is that not only are the ideas themselves interesting and beautiful, but the protagonists of the stories, the two women who had these ideas, are even more fascinating. So, in the next 25 minutes, we're going to turn back time, and tell the stories of two incredible mathematicians, and their observations about symmetry, that made us understand better the world we live in today. Are you ready? Okay, so at this point in the talk, I'm going to start talking about math. You shouldn't be surprised. You are the ones who decided to come to a public math lecture in a bar. But I really want you to understand the math that I'm talking about. And to understand math, you have to engage with it. You can't just let it wash over you, or nothing's going to sink in. So, in this first part of the talk, I ask that you engage with me. And in particular, I ask two things of you; the first is that you're willing to talk to strangers, and you'll work with the people around you. And the second, is that you don't get frustrated if the problem isn't clear to you right away. After all, you have the rest of your life to think about it [laughter]. So together, we're going to play a game. We're going to play this game three times. And each time we're going to bump things up a dimension. The first time I'll play the game in front of you, and explain to you how the rules work. The second time, you'll play the game yourself. The third time, well, we'll see how we go. So, the first round of the game goes like this. Well, all rounds of the game go like this actually. I have game pieces, and I have rules. And what we're trying to do is build something with the game pieces, while still following all of the rules. Anything we can build with the game pieces, which doesn't break any rules, is called a solution to the game. And together, what we're trying to figure out, is how many solutions the game has in each dimension. So, here's the one-dimensional version. My game pieces are these; a bunch of sticks, all the same length, as many of them as I want. And my goal is to put these sticks together in some way, by glueing them end to end, to create a shape that can live flat on a table. But in my gluing, I have to follow two rules. The first rule is that at every point where I glue two sticks together, the angle has to be the same. The second rule is that my shape has to close back around on itself. The beginning of the first stick, has to be connected to the end of the last stick. Here's an example. With three sticks, I could glue them together at tix degree angles and make a triangle. This satisfies all the rules because I could lay it flat in a table, and at – for every pair of sticks, there's a 60-degree angle between them. And also, it closes all the way back around itself. A solution, what else could I make? Well, with four sticks, I could put them together at 90-degree angles and make a square. With five sticks at 108-degree angles, I could make a pentagon. With six sticks, I could make a hexagon, or a heptagon with seven sticks, an octagon with eight sticks, a nonagon, a decagon, an 11-gon, at some point in time I lose the cool names. I could use 15 sticks and make a 15-gon. So, remember that our goal is to figure out how many solutions our game has. So, in this first version, how many solutions are there? Yes, there are infinitely many solutions. Any regular polyhedron, like the triangles and squares, with any number of sides, satisfies all of our rules. So, it's the solution of the game. Great. So now it's your turn. In the second round of the game, the game pieces are now the solutions of the previous round of the game. Triangles, squares, pentagons hexagons, hexagons, 15-gons. And what you want to do is construct some three-dimensional shape, using all one type. All triangles, all squares, all pentagons, all 15-gons, et cetera. But you have to satisfy two rules in your gluing. The first rule is, that at every corner of your shape, the shape has to look the same. And the second rule, is the shape has to close back around on itself. Every edge has to be connected to another edge. Here's an example; with four triangles. I could construct this shape; a tetrahedron. At every corner, I have three triangles meeting. So, with each of the four corners, the shape looks the same. A solution. Now it's your turn. I'm going to give you three minutes to work with the people around you, and see if you can find other solutions to the game. If you already know the answer, play anyway. Craft time is good for your soul. And if you don't know the answer, then talk to the people around you. Start now. So, I know that this talk is being made into a podcast. And it's maybe a little bit rude for me to leave the audience at home, sitting on their heels for three minutes and doing nothing. So, for the podcast audience, I'm going to narrate what's happening in this room. So, we're in an atmospheric bar called Harpoon Harry's and Surry Hills, in beautiful, sunny Sydney, Australia. And in this little room there are about 70 people. And on the tables in front of them, they have colourful piles of triangles, and squares, and pentagons, and hexagons, and hexagons. And as you can probably hear, they've taken my instructions to talk to strangers, very seriously. And as I talk, they're busily chatting away and building things out of all their shapes. Perhaps you played this game before, in primary school. It's a really fun one to play with kids. The ancient Greeks were also big fans of this game. They thought the solutions to this game were so beautiful, that they must be a fundamental fact about the universe. In fact, in 360 BCE, Plato wrote a dialogue called tumeo [phonetic]. Which was a long philosophical treatise about human nature and the world. And the culminating chapter of this book, was about the solutions to this game. Plato argued that since the solutions to this game are the most perfectly symmetric things that we can construct in three dimensions, they must be the fundamental building blocks of the universe. There were some notable flaws in Plato's theory. For example, he wanted to assign one of the four fundamental elements to a solution to the game, but earth wind, fire water, there are four fundamental elements and spoiler, there are more than four solutions to the game. So, this idea didn't quite work out. But Plato wasn't so far off. It turns out that these supersymmetric shapes, they do appear in nature. As people are probably experiencing right now, these things are a little bit hard to build at first. They're sort of floppy. But as the shape comes together and closes back around on itself, it becomes more rigid. This structural integrity is a good quality, if you are, say, a virus, who needs a strong protein shell. And sure enough, there are many examples in nature, of viruses constructing themselves exactly into these shapes, because of the structural integrity. And on a more philosophical level, Plato wasn't so far off in this idea that symmetry should underlie our physical worlds. One of the most profound observations of 20th century physics, is that symmetry is at the heart of physical law. For example, in quantum mechanics, the fundamental states of particles are given by their symmetry groups. So, Plato's idea that the world should be made from these supersymmetric solids, was, as actually exactly the right idea, just at the wrong scale. But now, I'm getting ahead of myself. We should bring ourselves back to Harpoon Harry's, here in Surrey Hills. Okay. I know that three minutes is not nearly enough time to play this game, and that you could continue building and building and you're having so much fun. But I have a lot of stories to tell you. So, it's time to wrap up. What have you built? Show me. I see one here. It's not the one I'm looking for, yet. Keep showing me more. That's the one I'm looking for. A cube. Yes. For those of you in the back who can't see, here's a big one. A cube is the second solution to our game. Made from six squares, with three squares meeting at each corner. Great. Okay. What else? Yes. Now Marcus. An octahedron. Here's the one that's slightly bigger than that one, so you can see it. This is made from eight triangles. It sort of, looks like two pyramids on top of each other. But if you turn it, you see that each corner has exactly four triangles meeting. So, it is perfectly symmetric. Awesome. We have three solutions. Is there anything else? Wow. I'm so impressed. So, this group in the back has very quickly, built this beautiful solution. From 12 pentagons, we can construct a dodecahedron. I'm very impressed. I really didn't think that anybody would be able to build that and only three minutes. So good work back there. So, our fourth solution to the game, is a dodecahedron. Cool. All my drinks are in the way. Anything else? Oh, you – oh, what did you discover? But remember my rules. My second rule is the shape has to close back around on itself. So, it's not allowed to go on forever. I could have restated my second rule, as you can only use finitely many things in your building. I say this is a debate for afterwards [laughs]. So those of you who had hexagons or heptagons on your table, might be a little frustrated, because you might have realised that building things from these, doesn't quite work. But there's one more maybe. There's one more solution that's very hard to build in three minutes. But if you had 20 you could have built an icosahedron, made from 20 – oh, it sounds like someone did good one. Good work. Yay. So, an icosahedron. Oh, wow. You all did very well. So, 20 triangles, with five triangles coming together at each corner, forms this shape, in icosahedron. Is this all? Well, maybe you're not convinced. But I claim that yes, these are actually the only solutions of our game. These are called the five regular solids. The tetrahedron, the cube, the octahedron, the dodecahedron, and the icosahedron. Sometimes they're called the five platonic solids for Plato's tumeo's [phonetic] chapter, that none of you heard me talk about. Because you were all so busy talking to each other, which is great. And I sort of hope, that you're not convinced that these are the only solutions. I spent a lot of time cutting out all these shapes. So, if you're not convinced, it's really fun to play around, and try to understand why you can't build anything else. So, if you're in that boat, take some home, and play at home, and convince yourself that these are really the only five solutions to this game. Great. Okay, so let's recap what we've discovered so far. In the first round of our game, we started with one dimensional game pieces, and we built two dimensional solutions. We found that there were infinitely many. In the second round of our game. We started with two-dimensional game pieces, and we built three dimensional solutions. We discovered that there were exactly five, the five regular solids. But I told you we were going to play three rounds of our game. So, what comes next? Well, we just keep going. In the next round of our game. The solutions are the – the game pieces are the solutions of the previous round. The five platonic solids are our game pieces. And now we're trying to glue them together in some way, to construct a four-dimensional shape, that satisfies all of our rules as before.

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>> What? How can we do this, right? We don't have a fourth dimension to work with. Well, from a mathematical perspective, there's no difference in going between one dimension to two, from two dimensions to three, from three dimensions to four. Just because we can't actually hold the solutions in the dimension that we live in, doesn't mean that the game doesn't make sense. We can still imagine and use our minds to figure out how many solutions the game has to have. But I'm not going to tell you the answer just yet. Instead, I'm going to take you to the year 1860, in the city of Cork, Ireland. Mathematician George Boole, and his wife Mary Everest, have just given birth to their third daughter, Alice. Four years and two daughters later, George dies, leaving Mary Everest with five young daughters, and no income. So, she needs to support her family. So, she packs up all the girls and moves everybody to London. And she takes a job as a librarian, at the Queens College of London. The girls grow up in their mother's library. The family has no money for a formal education for the girls. But Mary Everest had been a mathematics student of George before he died, and she had a lot of thoughts about mathematics education. She'd even written several books on the subject. Asserting instructions such as, the geometric education, can begin as soon as the child's hands can grasp objects. Let her have, among her toys, the five regular solids and a cut cone. So, we can imagine little Alice sitting on the floor of her mother's library, playing with these five regular solids, that are currently littering the tables of this bar in Surry Hills. In 1878, when Alice was 18 years old, her older sister, Mary Ellen, married an amateur mathematician and school teacher, named Charles Howard Hilton. At this time, Charles was obsessed with the idea of the fourth dimension. And he had set about on the task of teaching himself and others to visualise it, using a rigorous training procedure, involving hundreds of small cubes, that he built himself. He was even writing a book on his theory, and he decided that it'd be useful to test out his methods on some available subjects; his sisters in law. So, every time he went over to the Boole house for dinner, he would bring along his collection of cubes and set the five Boole daughters, Mary-Ellen, Margaret, Alice, Lucy, and Ethel, on the task of building things with these cubes, and on the task of memorising the arbitrary list of Latin names that he assigned to them. Most of the girls found this task terribly boring,

but it caught Alice's attention. She realised that using Charles's methods, she could think about four dimensional versions of the three-dimensional toys of her childhood. In the next 15 years, Alice's life continued as a life of any woman in late 19th century London would. She got a job as a secretary to support herself, got married to Walter Stott in 1890, and she had two children. But all the while she was still thinking about the fourth dimension. She realised that using Charles's methods, she could construct four dimensional analogues, to each of the five regular solids. From five tetrahedrons, she could build a sort of hyper tetrahedron, in four dimensions. From eight cubes, she could construct a hyper cube. There was an analogue of the octahedrons, which was built from 16 tetrahedrons, from 120 dodecahedron, she could build a sort of hyper dodecahedron. And there was even a four-dimensional analogue of the icosahedron, built from 600 tetrahedrons. So, it was – the year was 1895 and Alice had solved our game. She figured out – oh, sorry, she hadn't quite solved our game yet. The punch line hasn't come. So, what Alice realised, was that there were analogues [inaudible] the five platonic solids, but that there was one more shape that could she could construct in four dimensions. She could build from 24 octahedrons, an extra solution to the game, which had no analogue in three dimensions. So now the year was 1895, and she had solved our game. She found that there were exactly six solutions in four dimensions. So, at this point in time, Alice had uncovered something amazing. Let's take a second to look at her life. Working with no contact with the outside mathematical world, with no formal education, and purely out of her own curiosity, she had uncovered an exceptionally beautiful idea; which is, at least with respect to this game, the fourth dimension allows for more symmetry, than the third. This is the first of the beautiful ideas, that I promised you when we started today. And it's worth taking a moment to pause and consider it. Remember that in two dimensions, our game had infinitely many solutions; the regular polyhedral. In three dimensions, our game had exactly five solutions. The five regular solids are the only solids that admit this maximal amount of symmetry. But in six dimensions – sorry, four dimensions, with that one extra dimension of wiggle room, there are six solutions. So, you might think that every time we add a dimension, we should add more wiggle room and we could find more solutions. But surprisingly, this isn't what happens. In seven of dimensions – or I'm not counting very well, am I? In five dimensions, there are three solutions. In six dimensions, there are three solutions. In seven dimensions, there are three solutions. And in every dimension past seven, we will be stuck forever at just three solutions. So, this idea is really very amazing. That, at least with respect to this game, four dimensions allows for the most symmetry. So just past our physical world, in that one extra dimension, we find all of these extra symmetries. Let's return to Alice. In 1895 – my dates are probably a little bit wrong here. But soon after she made this discovery, somebody told Alice about a mathematics paper, written by a professor named Pieter Schoute at the University of Groningen, in the Netherlands. In this paper, there were – this paper was written in Dutch, first of all, so probably Alice couldn't understand it. But in this picture were drawings, that looked exactly like these three-dimensional cardboard cutouts that Alice had made, to describe her four-dimensional polytopes. Alice wrote to Professor Schoute, sending him pictures of her three-dimensional cardboard models. And he wrote back immediately with interest. Proposing that they work on a collaboration together. He came to England, and over the next 20 years, a fruitful friendship and collaboration grew. They published her findings on four dimensional polytopes, and other findings in four-dimensional geometry. And all the while she continued her life as a wife, a teacher, and a mother. So, in 1913, Schoute died. And in honour of their collaborations, the University of Groningen awarded Alice an Honorary Doctoral Degree in math. She was 53 years old. Meanwhile, across the English Channel, 31-year-old Emmy Noether was also thinking about symmetry, in the form of a highly esoteric thing called invariance of biquadratic forms. She had completed her PhD six years earlier at the University of her Erlangen in Bavaria. As a woman with a mathematics PhD at the turn of the century, Noether was a very unusual figure at this time. To get to this point in her life, she had already been one of only two women in her University of 950 students. And she had needed to obtain permission from her male professors, even to attend their classes. Women were not allowed to hold academic positions at this time. So, when Noether finished her PhD, she had stayed at the University of Erlangen, and worked without pay for the last six years. But the next year, she received an invitation from mathematician, David Hilbert and Felix Klein at the University of Göttingen. They invited her to come and work there. Unfortunately, they encountered an obstacle in their invitation. Hilbert and Klein had wanted Noether to come, because they had recently heard about Albert Einstein's theory of general relativity. Which was not yet announced, but rumoured to be almost complete. And they were determined to test its validity. So, they wanted to have Noether around because of her expertise in invariant theory. But they had an obstacle when they tried to bring her to Göttingen. The Faculty of Philosophy at the University of Göttingen, denied her candidacy for professorship, because she was a woman. Though Hilbert and Klein tried hard to support her case, Hilbert famously argued, this is a university, not a bathing establishment. The faculty wouldn't budge. But Noether was used to working without pay. So, she came to go to Göttingen anyway. And one month later, Einstein arrived to give a series of six lectures on his theory of general relativity. During these six lectures, Einstein managed to convince Hilbert and Klein that his theory was solid, and three months later, he published the famous paper, announcing general relativity. So, by the end of 1915, the theory of general relativity was complete. But it had some odd qualities that made people uncomfortable, and left some with lingering doubt. Most notably, the theory did not satisfy local conservation of energy. The familiar notion that energy cannot be created nor destroyed, merely changes form had been a guiding principle on physics up to this point in time. Physicists didn't fully understand it. But the fact that Einstein's theory didn't satisfy this, was unsettling. Hilbert posed this energy paradox aspect of general relativity to Noether, and asked her if she had any idea what was going on. Three months later, she came back with an explanation not only for Gilbert's energy paradox, but also a much more general and fundamental observation about the relationship between symmetry and physical conservation loss. This is the second of the beautiful ideas that I promised you when we started today. And it's captured in the content of Noether's Theorem. Noether's Theorem states, that in any physical theory, every symmetry of the system, must have a corresponding conserved quantity. What does this mean? Well, physical theories such as general relativity, are dictated by a set of equations. And equations can have symmetry. For example, think about the Pythagorean equation relating the side length of a right triangle. [Variable] a squared, plus b squared, is equal to c squared. If we swap the role of a and b, we get the equation, b squared, plus a squared, is equal to c squared. This is the same equation we started with. Switching a and b doesn't change the equation. So, we say that this equation is symmetric, under the operation of swapping a and b. The symmetry makes sense when we – physically, when we remember that this equation is describing side links, and a triangle. If we switch the role of a and b, we get a different triangle, but the length of the hypotenuse doesn't change. So Noether's theorem, related the symmetries that occurred within the equations of a system, to physical quantities such as energy and momentum. For example, Noether's theorem states that in any physical theory that has spherical symmetry, meaning that rotation doesn't change the outcome of an experiment, this theory should have conservation of angular momentum. And in any physical theory, with translational symmetry – meaning that if I do an experiment here, or I do an experiment here, this shouldn't change the outcome. This theory should have conservation of linear momentum. And any physical theory with time symmetry, meaning that the theory is the same now as it will be in 20 years, should have conservation of energy. But Einstein's theory of general relativity didn't satisfy time symmetry in the same way as previous series had. It was much more subtle. So nervous explained that we shouldn't expect it to have conservation of energy, in the same way that other theories have. Noether's theorem is a deep observation about the relationship between symmetry in our physical world. And it's changed the way that we think about physics to this day. Noether's theorem shows us that hidden symmetry is not just a beautiful curiosity to think about, but that its existence, fundamentally dictates the framework of our universe. So, you'll be happy to know that eventually, Noether did get paid. After – but it wasn't until the end of World War One and the German revolution of 1918, that she was finally allowed to start her candidacy for professorship in 1919. She got paid for the first time in her life in 1923, when she was 41 years old. Noether's theorem was just one of her many contributions to mathematics and physics. She went on after this to make groundbreaking contributions, in Ring theory, Galois theory, and in my own field, Representation theory. But now the sad part comes. But in 1932, her life collided with history. When, as a Jewish woman living in Germany, she was forced to leave her academic position by Hitler's Third Reich. As conditions in Germany grew increasingly violent, she was forced to leave the country. And she accepted a visiting teaching position at Bryn Mawr College in the United States. In America, she travelled back and forth between Bryn Mawr and Princeton; lecturing and working at both locations for three years, until she suddenly and tragically died from surgery complications in 1935. She was 53 years old. The same age as Alice Boole Stott was, when she was awarded her honorary doctoral degree, 22 years earlier. Each of these women spent their lives searching for hidden symmetry. Just as I did as a little girl, staring at almond groves from the backseat of my parent's car. But the ideas that emerged from their search, were not just the beautiful daydreams of a child, they were fundamental observations about the nature of our universe. So, I hope that you find these stories as beautiful and inspiring, as I do. And the next time you find yourself noticing a pattern in the world around you, and wondering if that pattern is part of some bigger picture, I encourage you to let your mind wander to that place. You never know where it might go. I'll leave you today with my favourite mantra, spoken in the words of Professor Jordi Williamson right here at the University of Sydney. Never underestimate symmetry. Thank you.

[Applause]

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