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Department of Defence
Science and Technology

Gaining advantage from Complexity in Defence: a new DST research initiative

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Joint & Operations Analysis Division

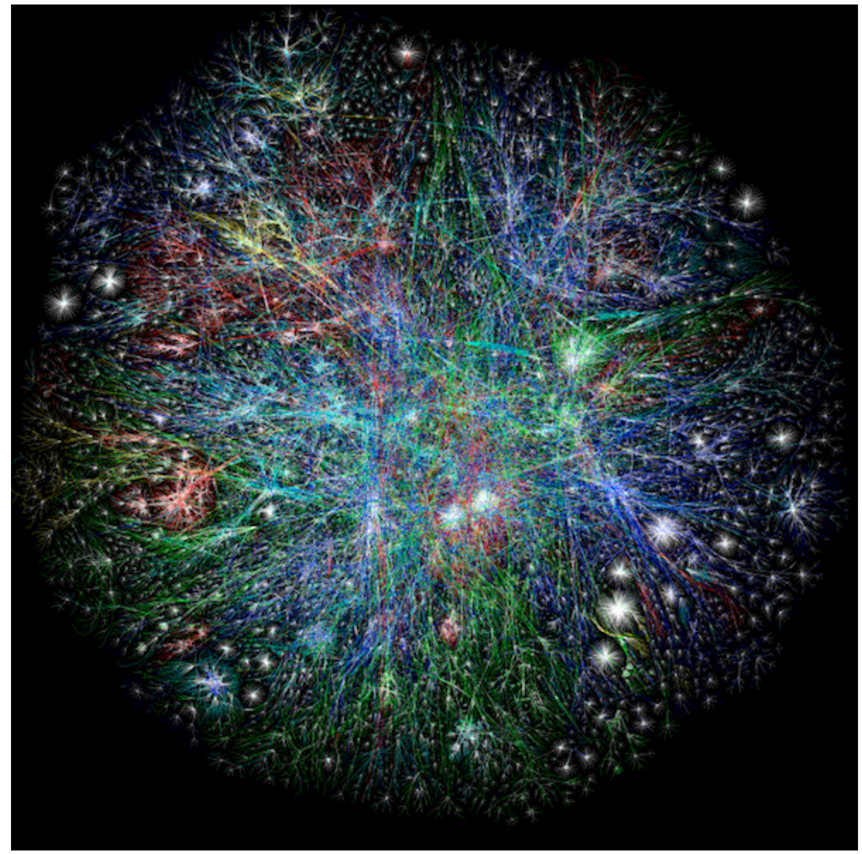
Outline

- Complexity – Good or Bad (for us)?
- DST Modelling Complex Warfighting SRI
- Models of Warfighting
- What’s missing?
- Decisions & Attrition: a ‘Kuramoto-Lanchester’ model
- Complexity advantage
- What are we looking for?
- Conclusions

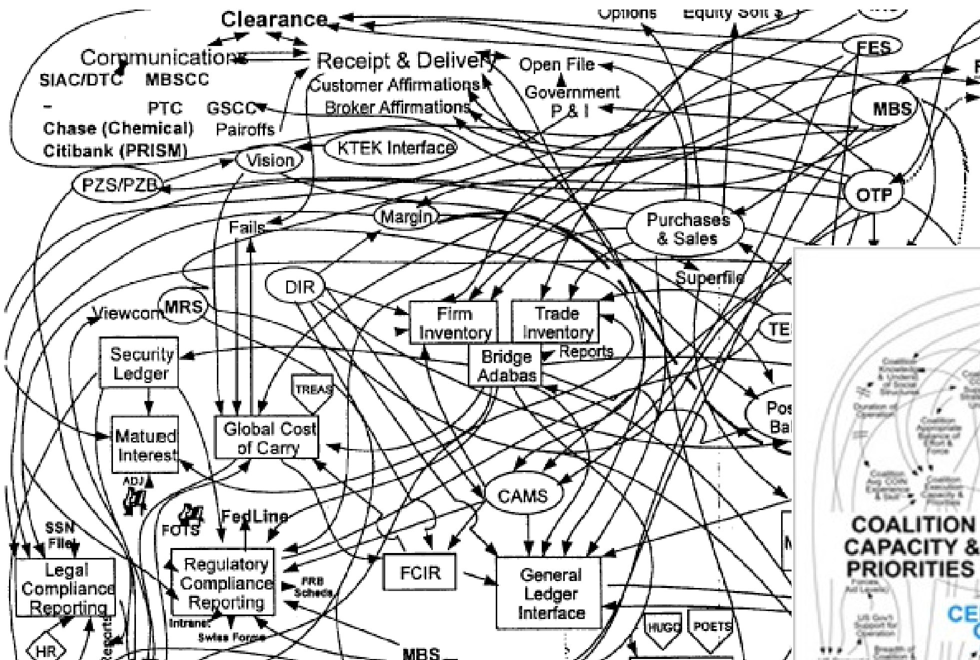
Complexity – Beauty …

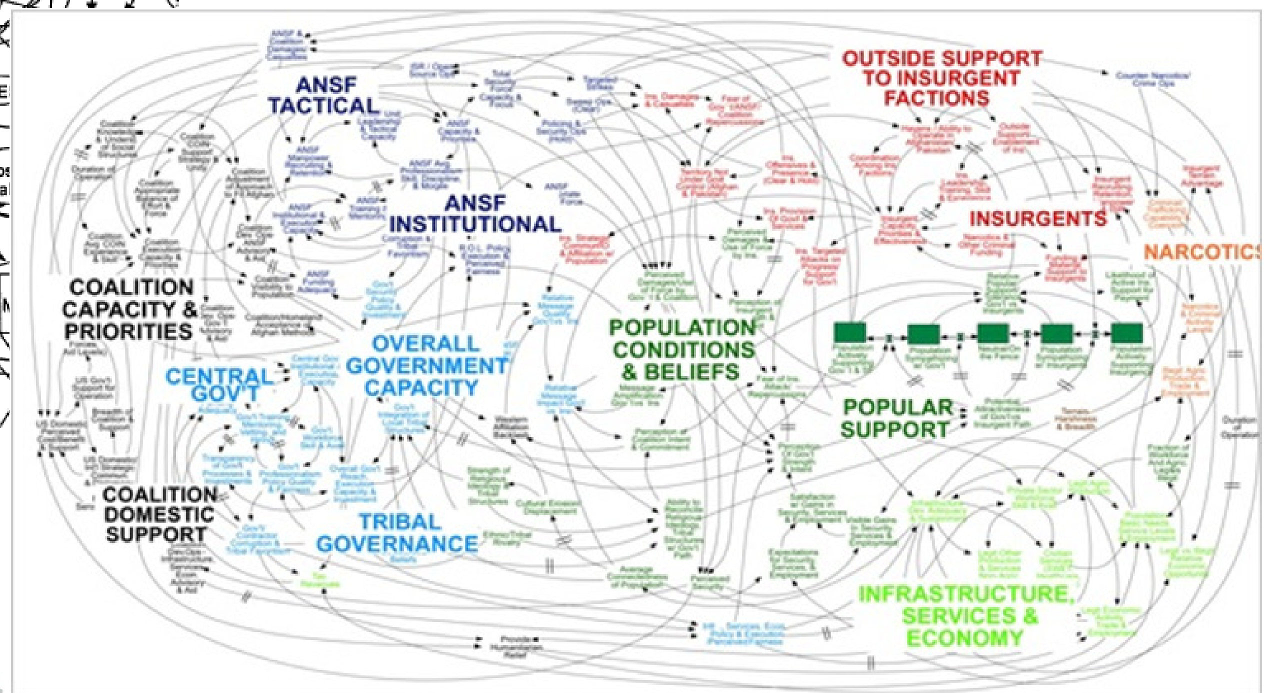
The Internet

Motor and Somatosensory Cortex



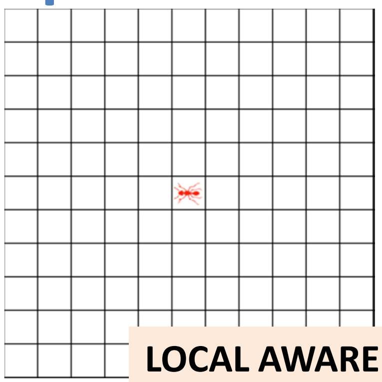
Complexity – and The Beast?

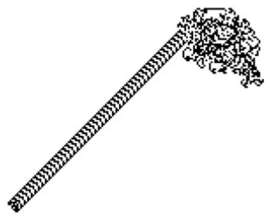


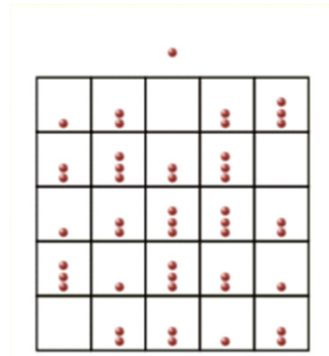


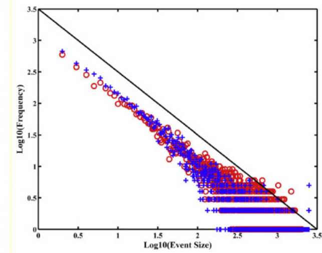
GROUP

Complexity ‘phenomena’

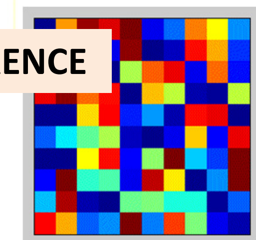


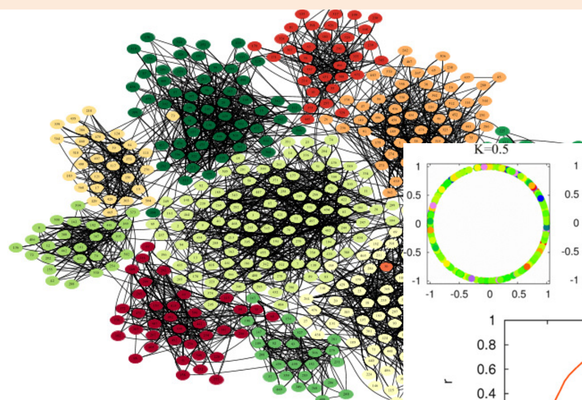


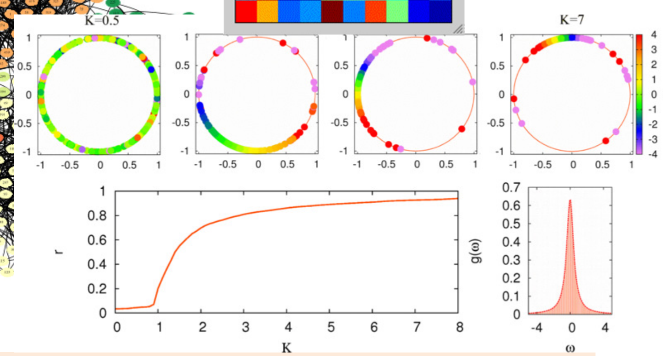




LOCAL AWARENESS/INTERACTIONS – GLOBAL COHERENCE







Can these properties be exploited to advantage in national defence?

The problem

- How can ‘emergence’/’criticality’/’self-synchronisation’/’self-organisation’ be exploited by a Force
  - To make it ***robust*** against shocks – ***resilience***
  - To give it ***advantage*** against a near-peer adversary
- We know some of the answers *in abstracto* – now is the time to see it for things that look like national defence.

Creating a Future Force: how DST supports Force Design

PREPARE AND EMPLOY THE CURRENT FORCE → CONNECTED THROUGH TIME → DESIGN AND DEVELOP THE FUTURE FORCE

- What are the parts?
- How do they connect?
- What else should we get?
- How do we gain advantage from them as a system for the future?

The future environment.
Models.
Data.
Simulation/Analysis.
Operating Concepts.

Modelling Complex Warfighting SRI*: *Revolutionising the analytical approach to force design*

*SRI = Strategic Research Investment

Conquering Uncertainty

Scientific methods to enable robust Force Design decisions to produce a resilient force through the understanding and management of uncertainty in Defence.

Innovative simulations

Novel modelling and simulation techniques to enable exploration of whole-of-force warfighting concepts and force options.

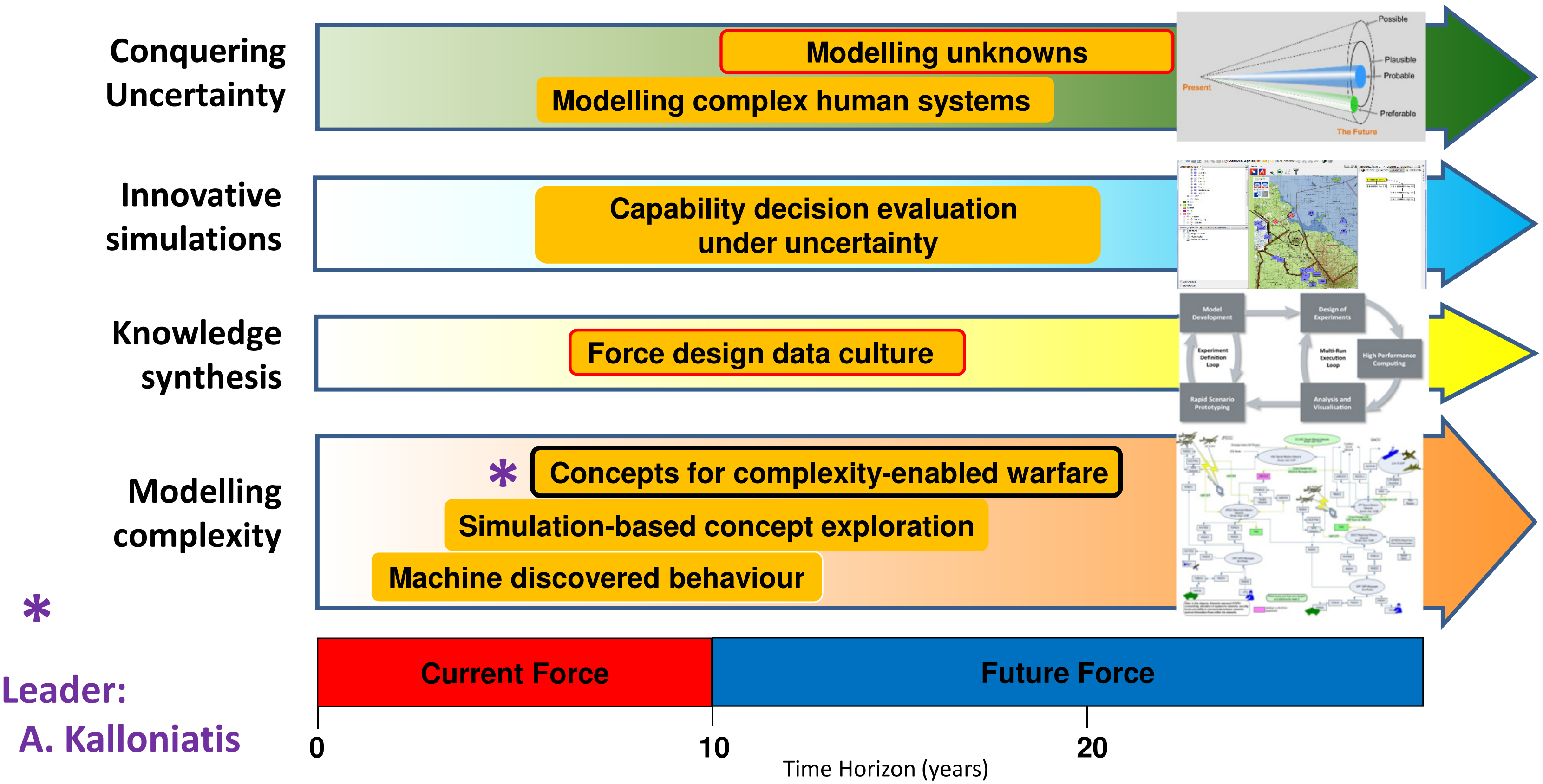
Knowledge synthesis

Synthesis of analytical and simulation results to support development of a joint force which is integrated by design.

Modelling complexity

Methods to enable understanding of properties of the joint force emerging as a result of nonlinear interactions between the many constituent elements.

Modelling Complex Warfighting SRI: *Revolutionising the analytical approach to force design*



Project Leader:
A. Kalloniatis

Mathematical Models of Warfighting

- Lanchester 1916 – (“Directed”) Force-on-Force Attrition

$$\dot{B}(t) = \alpha_S B(t) - \alpha_{RDA} R(t)$$
$$\dot{R}(t) = \beta_S R(t) - \beta_{BDA} B(t)$$

B= Size of Blue Force
R = Size of red Force

- Protopopescu et al 1989 – Diffusion, Advection, Inhomogeneity
- Hughes 1995 – Missile Salvos, Staying Power
- McLemore et al 2016 – Manoeuvre, Dispersion, Swarming, Swarming

What's missing – in one or another – or all?

- Logistics
- Deception/Reconnaissance
- Manoeuvre
- States of Readiness/Damage
- ***Command and Control (ie organisational decision-making) – hierarchical or networked***

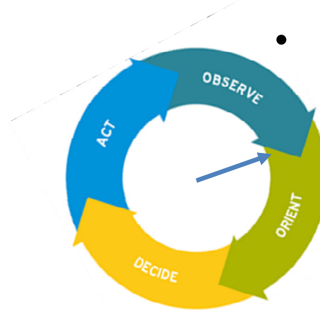
A model for C2 – the Kuramoto Model (1984)

Rate of progress through decision cycle

$$\dot{\beta}_i = \omega_i + \sigma \sum_j A_{ij} \sin(\beta_j - \beta_i)$$

Tightness of Organisational Coupling

Organisational Interactions



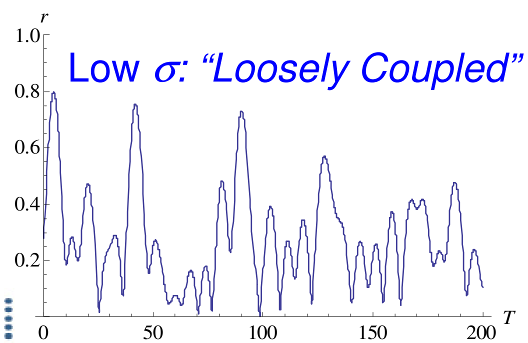
- β = state in continuous decision cycle

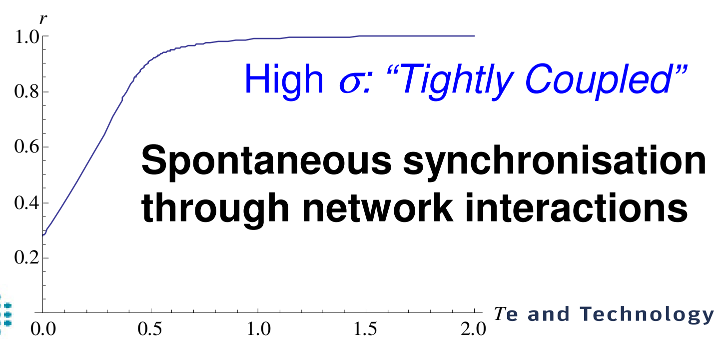
ω_i
Frequency of decision-making when left to self.

Socio/technical applications:
- Rhythmic applause (Neda et al 2000);
- Opinion dynamics (Pluchino et al 2006);
- Pedestrian crowds (Strogatz 2005);
- Decision making in animal groups (Leonard et al 2012);
- Planar vehicle coordination (Paley et al 2007);
- Control systems (Jadbabie et al 2004);
- Consensus protocol (Sarlette & Sepulchre 2009).

Measure of synchronisation:

$$r(t) = \frac{1}{N}\left|\sum_j e^{i\beta_j(t)}\right|$$





Spontaneous synchronisation through network interactions

External C2 driven resupply and symmetric direct attrition

- Kuramoto

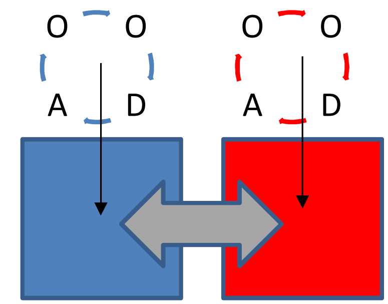
$$\dot{\beta}_i(t) = \omega_i + \sigma \sum_j B_{ij} \sin(\beta_j(t) - \beta_i(t))$$ Blue C2 system

$$\dot{\rho}_i(t) = \nu_i + \sigma \sum_j R_{ij} \sin(\rho_j(t) - \rho_i(t))$$ Red C2 system

- Order parameter

$$r_B(t) = \frac{1}{N}\left|\sum_j e^{i\beta_j(t)}\right|$$

$$r_R(t) = \frac{1}{N}\left|\sum_j e^{i\rho_j(t)}\right|$$



C2 capability ***sits outside*** the combat force

- Lanchester

$$\dot{B}(t) = r_B(t)B(t) - r_R(t)R(t)$$

$$\dot{R}(t) = r_R(t)R(t) - r_B(t)B(t)$$

Resupply

Attrition

Good C2 $\Rightarrow$ Good resupply of own and good firepower on adversary

Resupply and internal C2-direct attrition

Attrition undermines ability to couple on the network

- Kuramoto

$$\dot{\beta}_i(t) = \omega_i + \frac{B(t)}{B(0)} \sum_j B_{ij} \sin(\beta_j(t) - \beta_i(t))$$

Blue C2 system

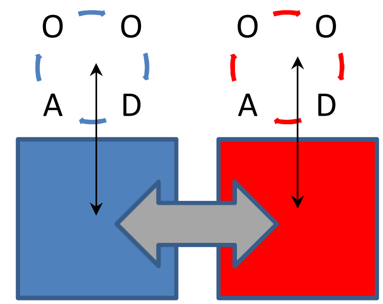
$$\dot{\rho}_i(t) = \nu_i + \frac{R(t)}{R(0)} \sum_j R_{ij} \sin(\rho_j(t) - \rho_i(t))$$

Red C2 system

- Order parameter

$$r_B(t) = \frac{1}{N}\left|\sum_j e^{i\beta_j(t)}\right|$$

$$r_R(t) = \frac{1}{N}\left|\sum_j e^{i\rho_j(t)}\right|$$



- Lanchester

$$\dot{B}(t) = r_B(t)B(t) - r_R(t)R(t)$$

$$\dot{R}(t) = r_R(t)R(t) - r_B(t)B(t)$$

Resupply

Attrition

C2 capability ***resides in*** the combat force

Good C2 $\Rightarrow$ Good resupply of own and good firepower on adversary

Resupply and internal C2-direct attrition

Attrition undermines coupling on and links of the network

- Kuramoto

$$\dot{\beta}_i(t) = \omega_i + \sigma_B \frac{\chi_B}{N_B} \sum_j \mathcal{B}_{ij}(t) \sin(\beta_j(t) - \beta_i(t))$$ Blue C2 system

$$\dot{\rho}_i(t) = \omega_i + \sigma_R \frac{\chi_R}{N_R} \sum_j \mathcal{R}_{ij}(t) \sin(\rho_j(t) - \rho_i(t))$$ Red C2 system

- “Order” parameter

$$\chi_{\mathrm{B}}(t) = \left| \sum_j e^{i\beta_j(t)} \right|$$

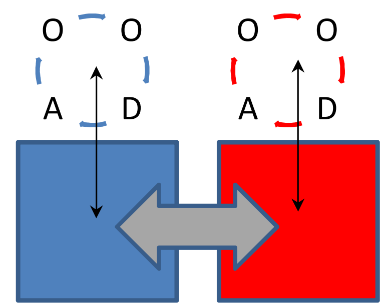
$$\chi_{\mathrm{R}}(t) = \left| \sum_j e^{i\rho_j(t)} \right|$$

- Lanchester

$$\dot{B}(t) = -\kappa_{\mathrm{R}} \chi_{\mathrm{R}}(t) + \eta_{\mathrm{B}} \chi_{\mathrm{B}}(t)$$

$$\dot{R}(t) = -\kappa_{\mathrm{B}} \chi_{\mathrm{B}}(t) + \eta_{\mathrm{R}} \chi_{\mathrm{R}}(t)$$

Attrition Resupply



C2 capability ***is identical to*** the combat force

Detecting criticality

- Kuramoto order parameter

$$r_B(t) = \frac{1}{N}\left|\sum_j e^{i\beta_j(t)}\right|$$

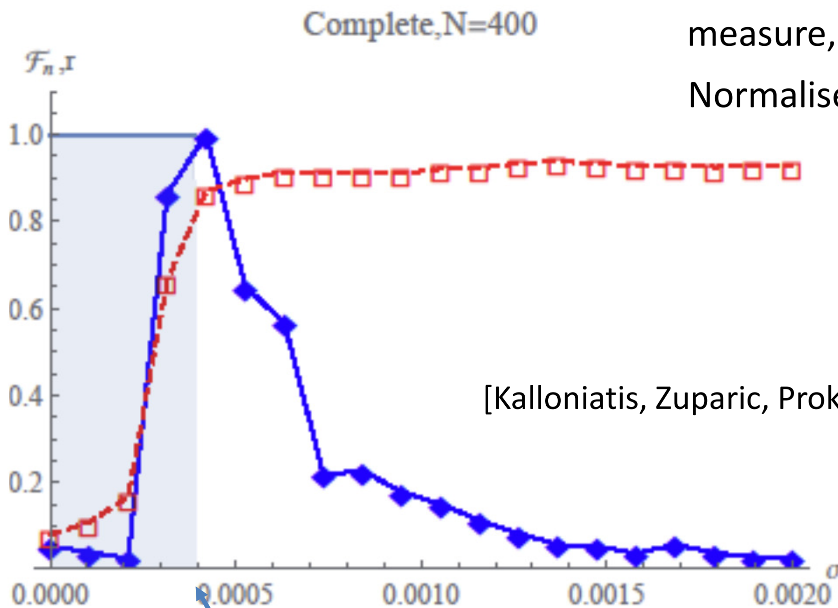
- Fisher information

$$\mathcal{F} \equiv \mathbf{E}\left[\left(\tfrac{\partial}{\partial\sigma}\log P(X;\sigma)\right)^2 \middle| \sigma\right]$$
$$= \prod_i \int dX_i P(X_i;\sigma)\left(\tfrac{\partial}{\partial\sigma}\log P(X_i;\sigma)\right)^2$$

$$\mathcal{F}_n \equiv \frac{\mathcal{F}}{\mathcal{F}_{\max}} \leq 1$$

Time-averaged synchronisation measure, *r*

Normalised Fisher information



[Kalloniatis, Zuparic, Prokopenko – PRE subm.]

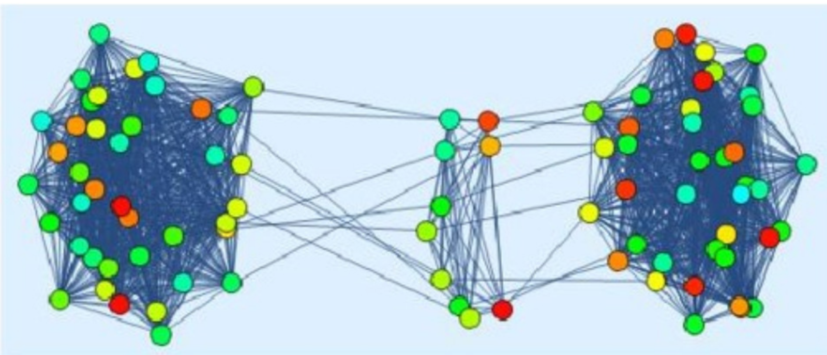
$$K_c = \frac{2}{\pi g(\varpi)}$$
$$\Rightarrow \sigma_c = 0.000398$$

(Kuramoto, 1984)

- Minimum description length

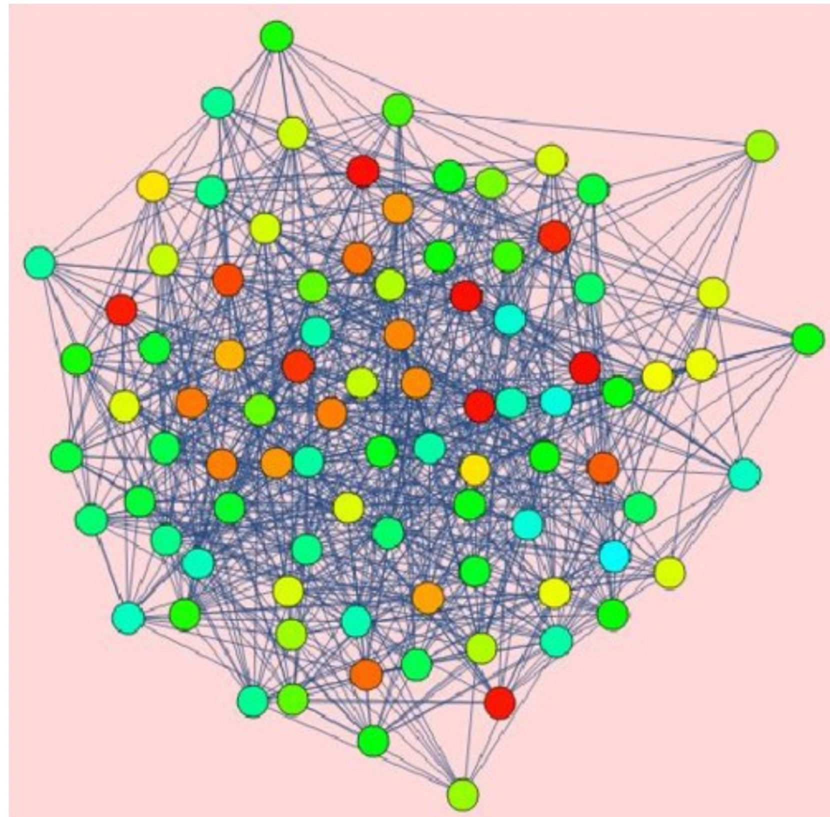
Proxy: in numerical solution, the minimum number of points required to describe time-series for a given value of coupling.

A Scenario



Blue – pseudo-hierarchical – headquarters entity covering two task groups of complete networks

Red – pseudo random network



Strategy:

1. **Solve ordinary Kuramoto dynamics for criticality indicators as function of coupling**
2. **Solve Kuramoto-Lanchester dynamics with static network**
3. **Solve Kuramoto-Lanchester with attrition of network**

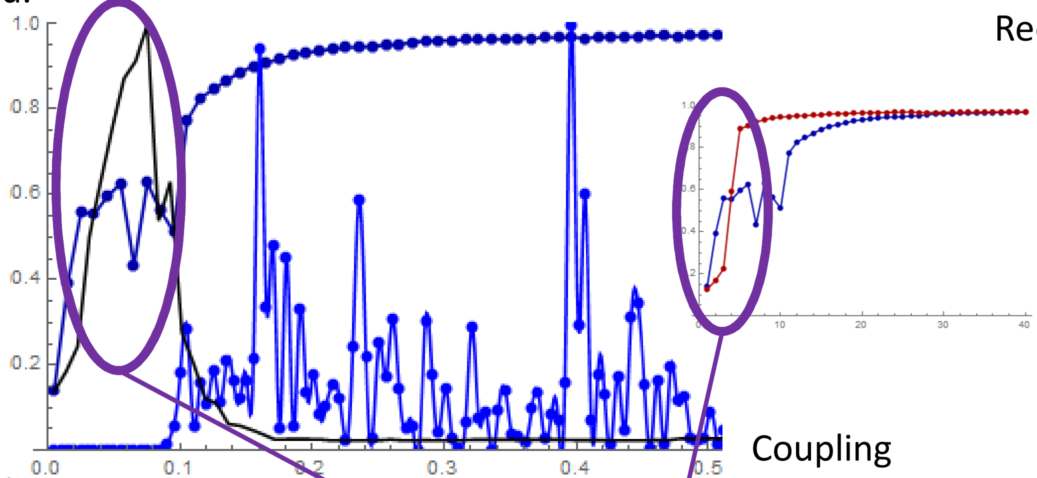
Does (1) give insight into (2) and (3)?

Complexity Advantage ?

Three measures of Criticality

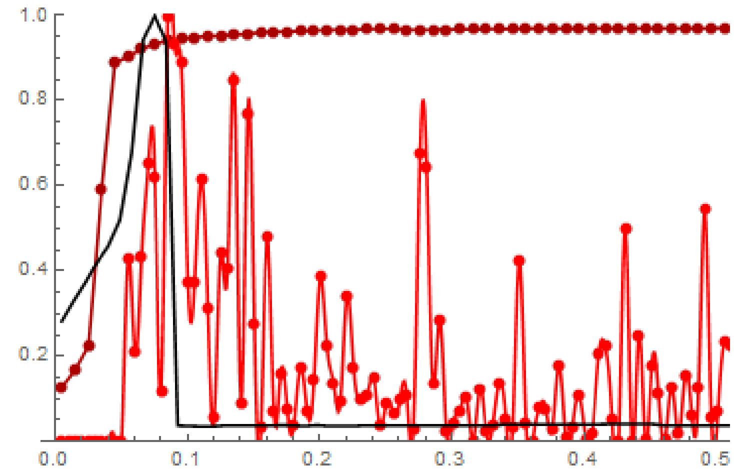
1.

Crit. Ind. Blue



Coupling

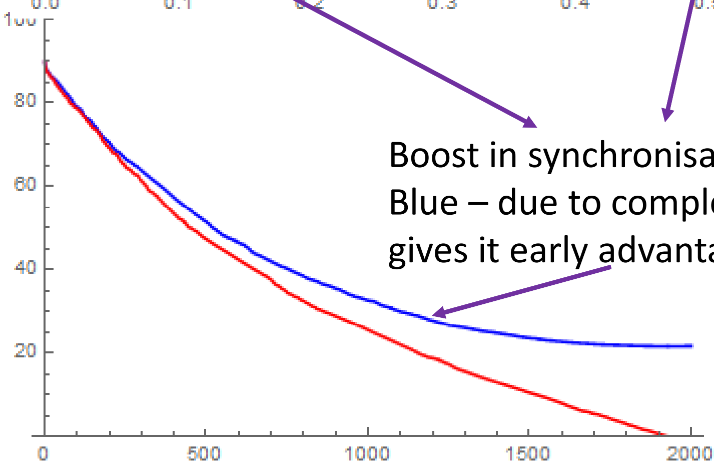
Crit. Ind. Red



Coupling

2.

B,R



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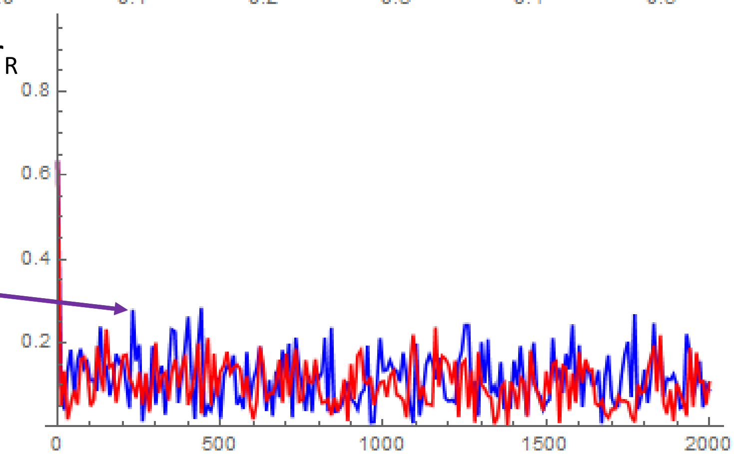
r_B, r_R



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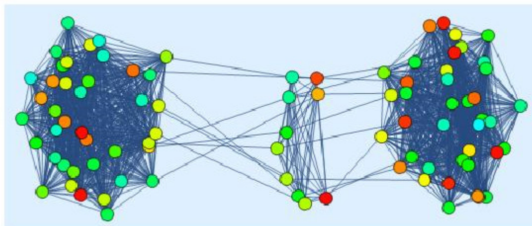
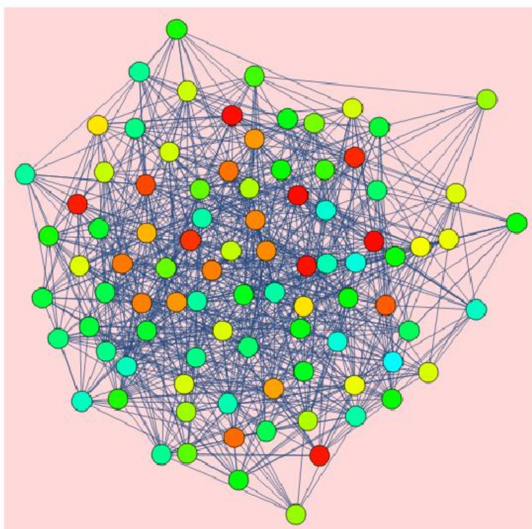
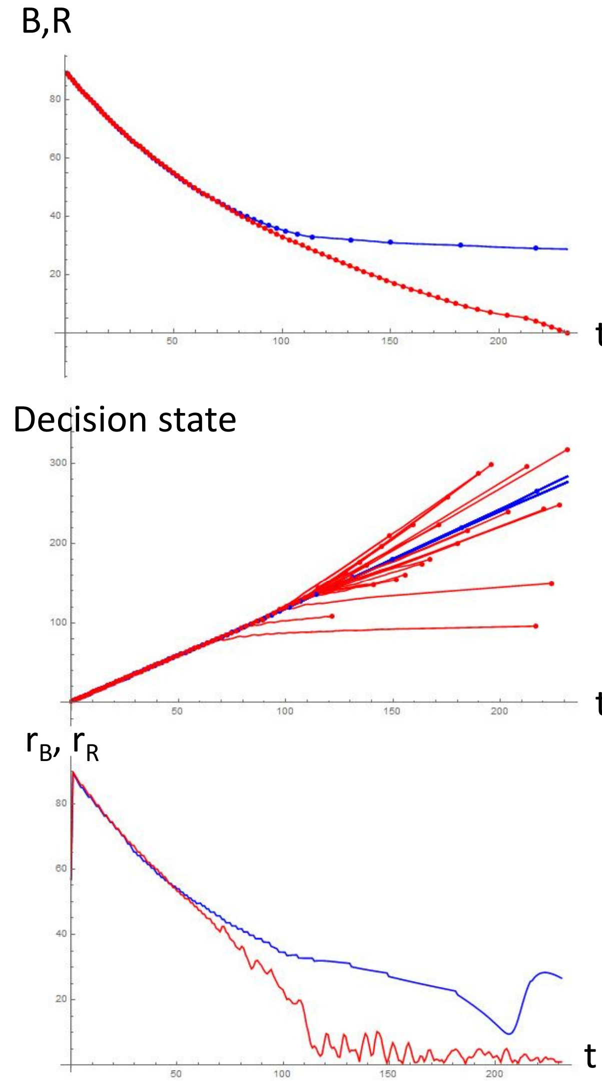
Boost in synchronisation at low coupling for Blue – due to complete graph connectivity – gives it early advantage that is maintained

3. Attrition of networks
- HQ 'protected'







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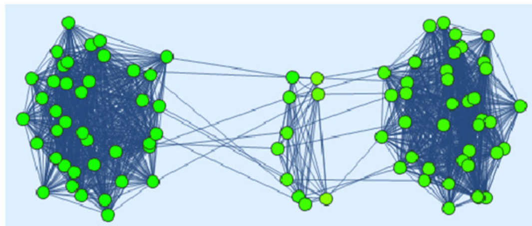
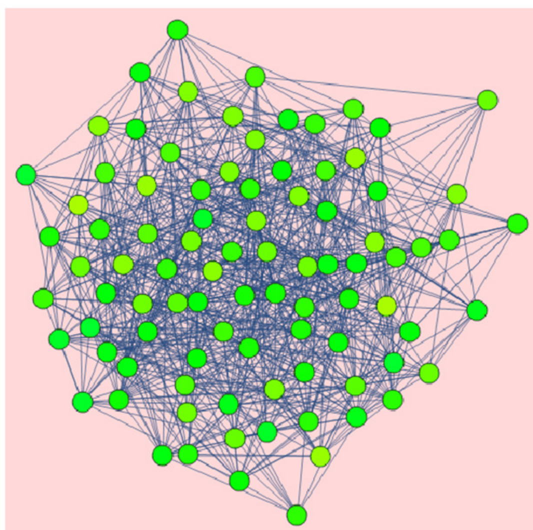
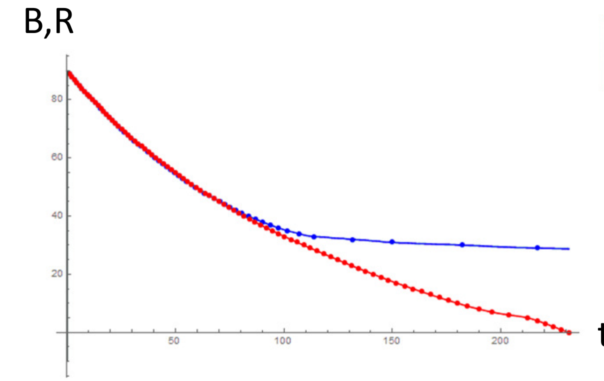
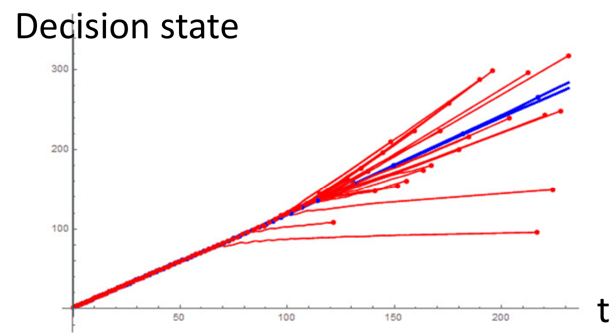
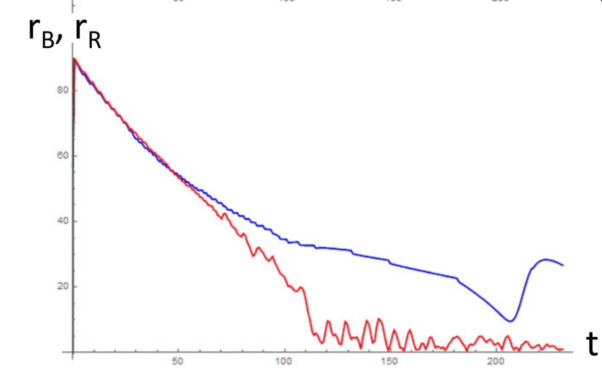


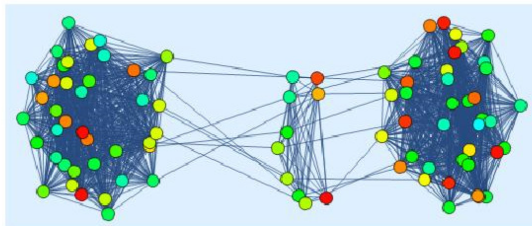


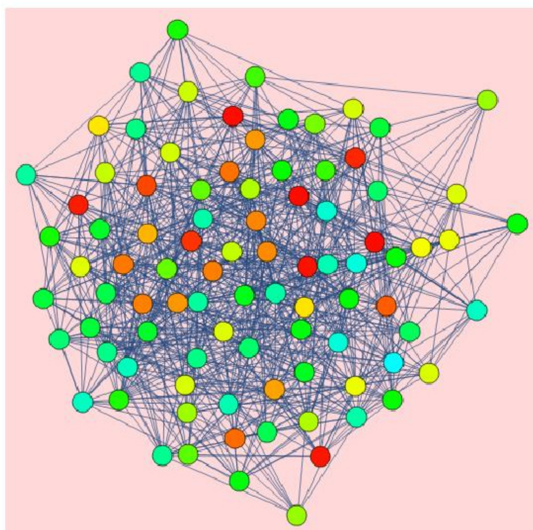


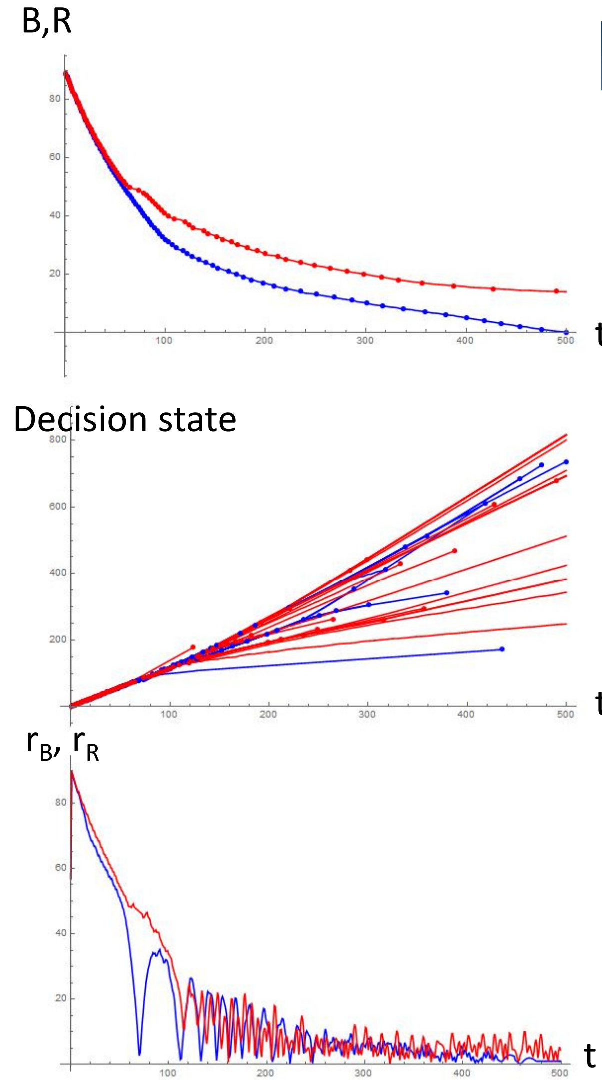




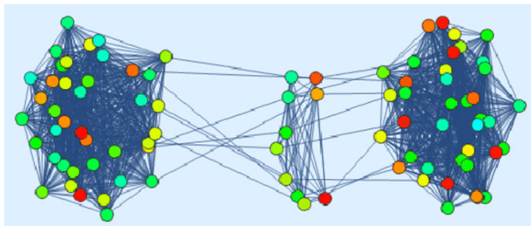
3. Attrition of networks
- HQ 'unprotected'

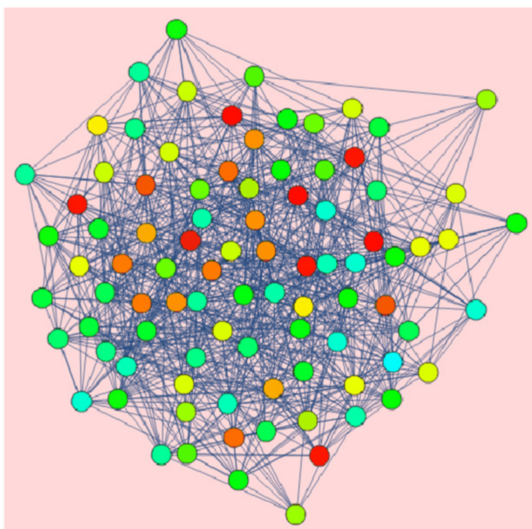


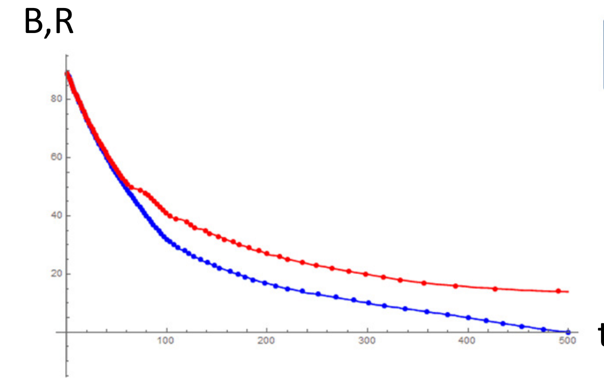


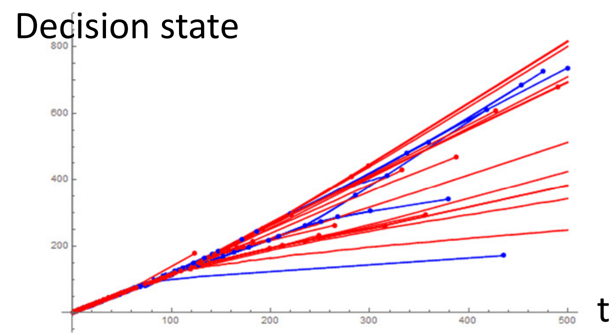


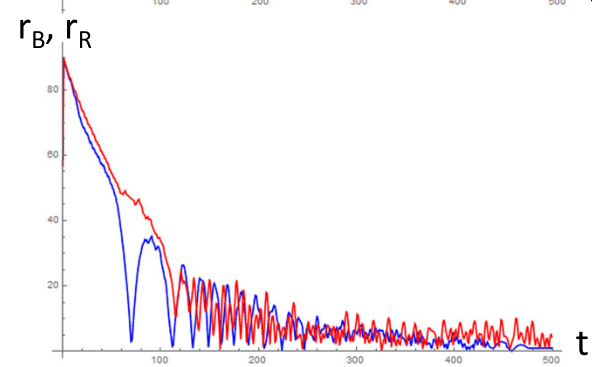
3. Attrition of networks - HQ ‘unprotected’



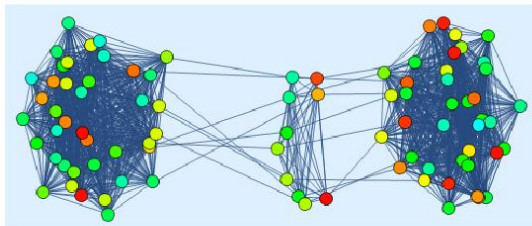




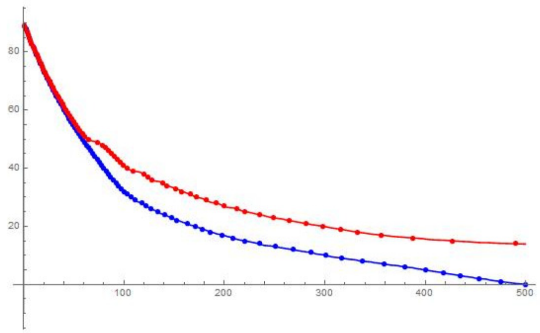




Attrition of networks
- HQ 'unprotected'



B,R



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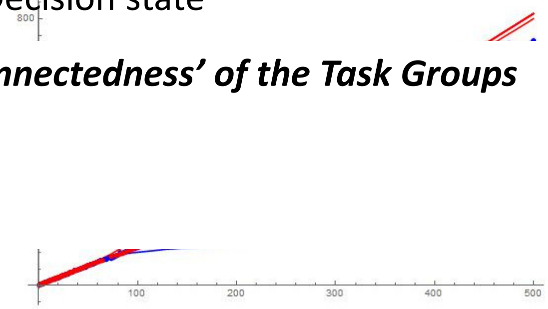
Decision state



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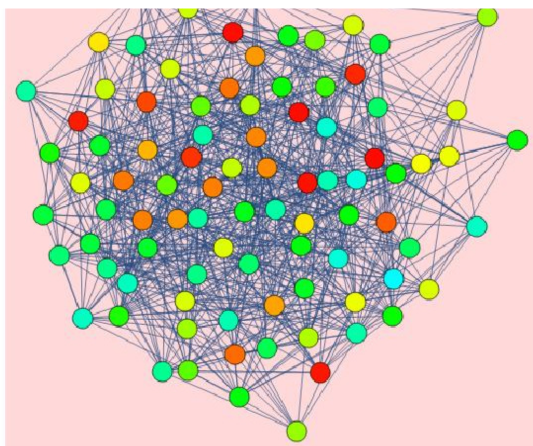
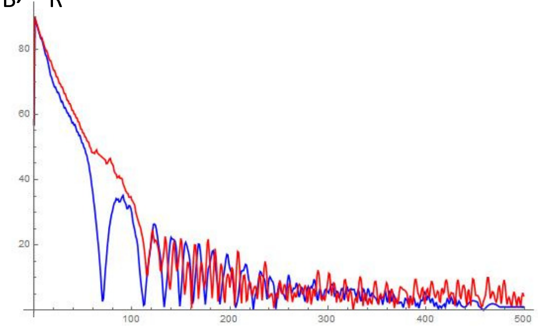
This is the more typical behaviour – unless the 'full connectedness' of the Task Groups is preserved, the 'boost in r' for Blue is lost.

Conservation of criticality?



r_B, r_R



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Early days …

- Approach to statistical limit – convergence of criticality indicators?
- Criticality indicators for dynamical network scenarios?
- Stochasticity, Advection – Gaussian and non-Gaussian
- Generalisation to more sophisticated representations of modern combat?

- ***Is concentration of mass/increase of number of actors the only way to achieve complexity/criticality?***

What are we looking for?

- Collaboration

- Just completed – initial Expression of Interest (EoI) process for start-up collaborations.

Modelling Complex Warfighting Symposium
Thursday 14th and Friday 15th December 2017
Victoria Division of Engineers Australia,
Bourke Place, 600 Bourke Street, Melbourne

- First of many …

Conclusion

- Complexity – feared but exploitable
- Marrying complex systems dynamical models with mathematical combat models enables generation of new warfighting concepts.
- New DST Strategic Research Initiative “Modelling Complex Warfighting” to pursue this.
- Opportunities for peer-to-peer collaboration with academic partners in ranges of areas:
  - Statistical physics
  - Network Theory
  - High Performance Computing
- Watch this space – or contact me …