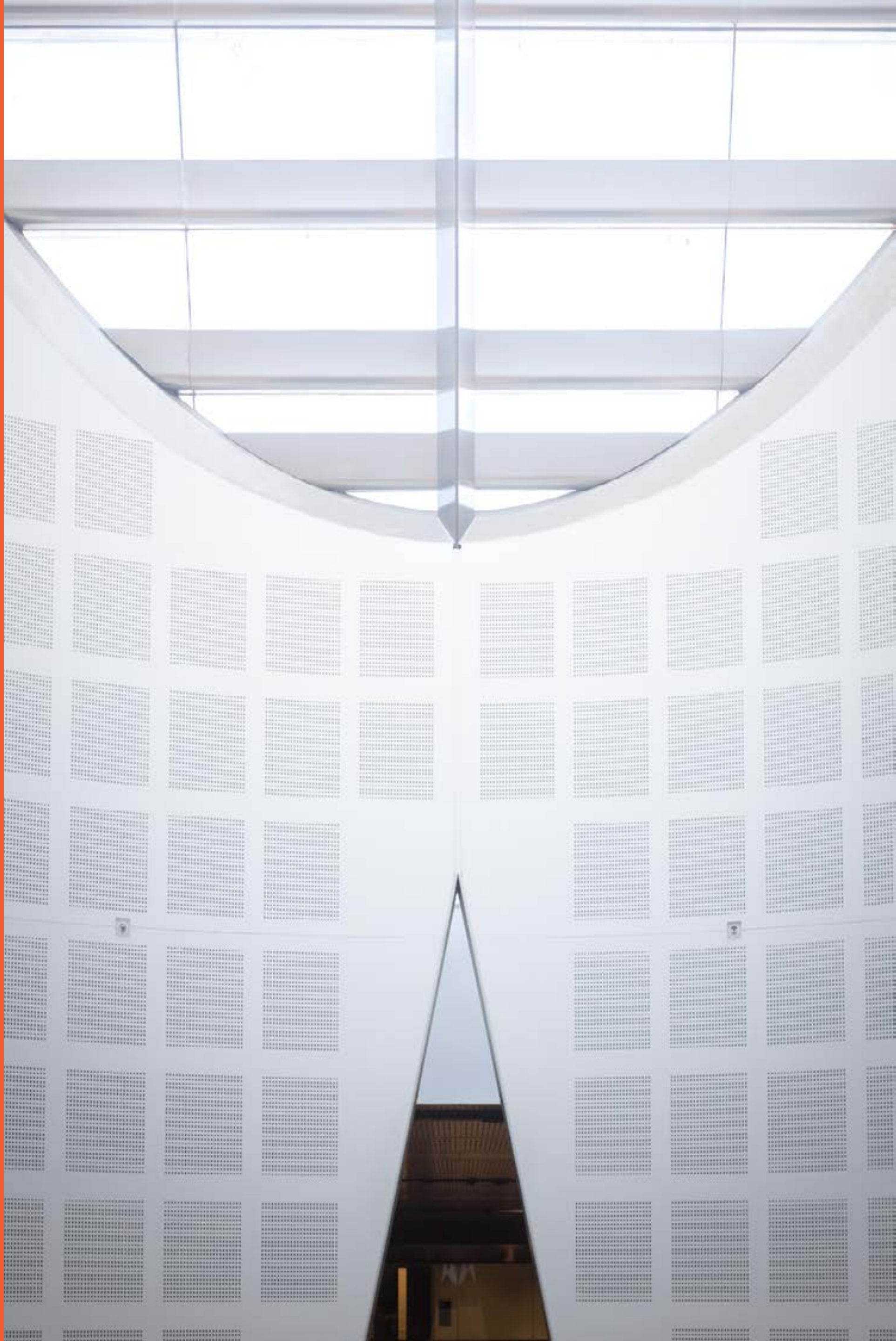
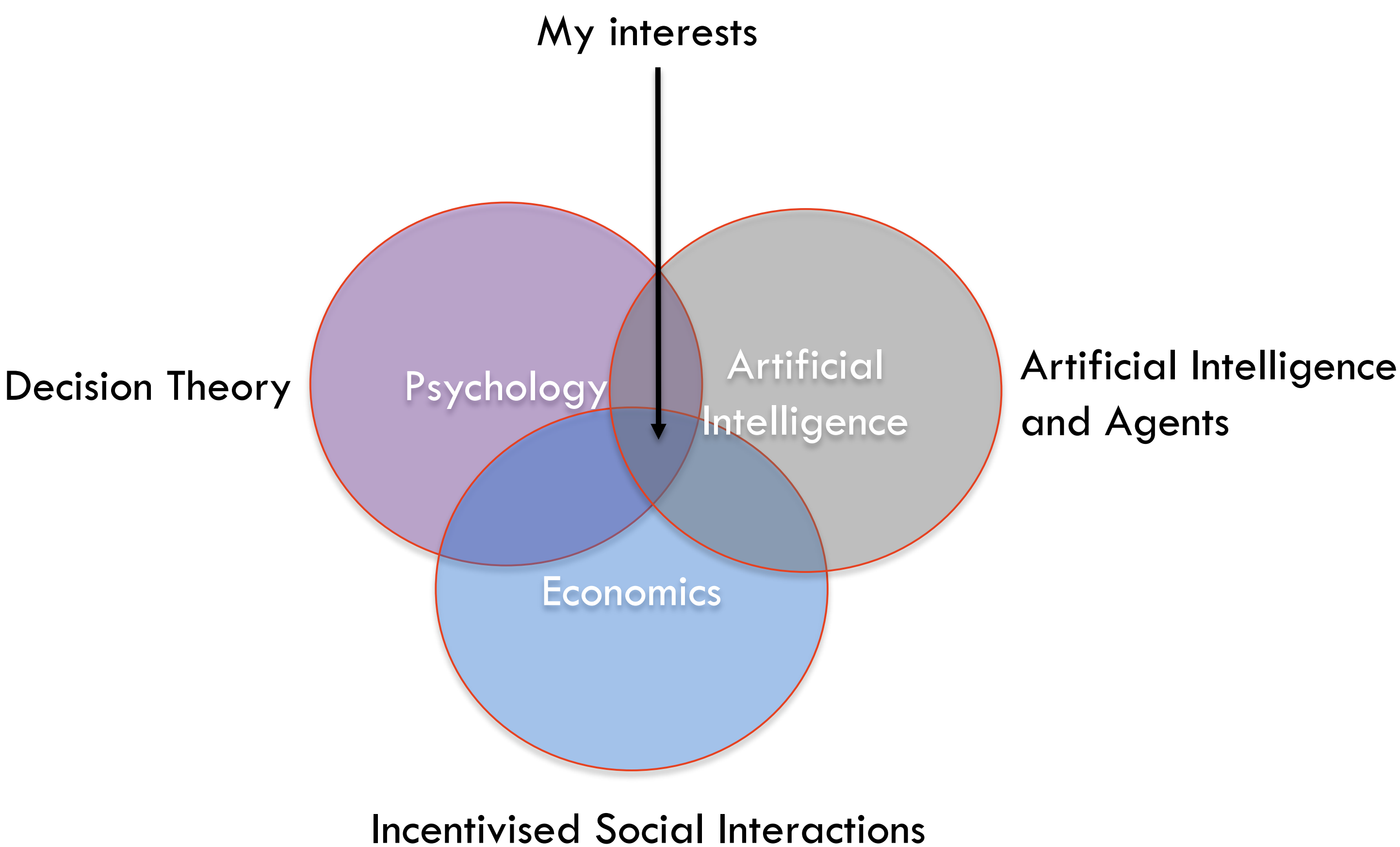
Economic Crises in Agent Based Models of Housing Markets

Michael Harre

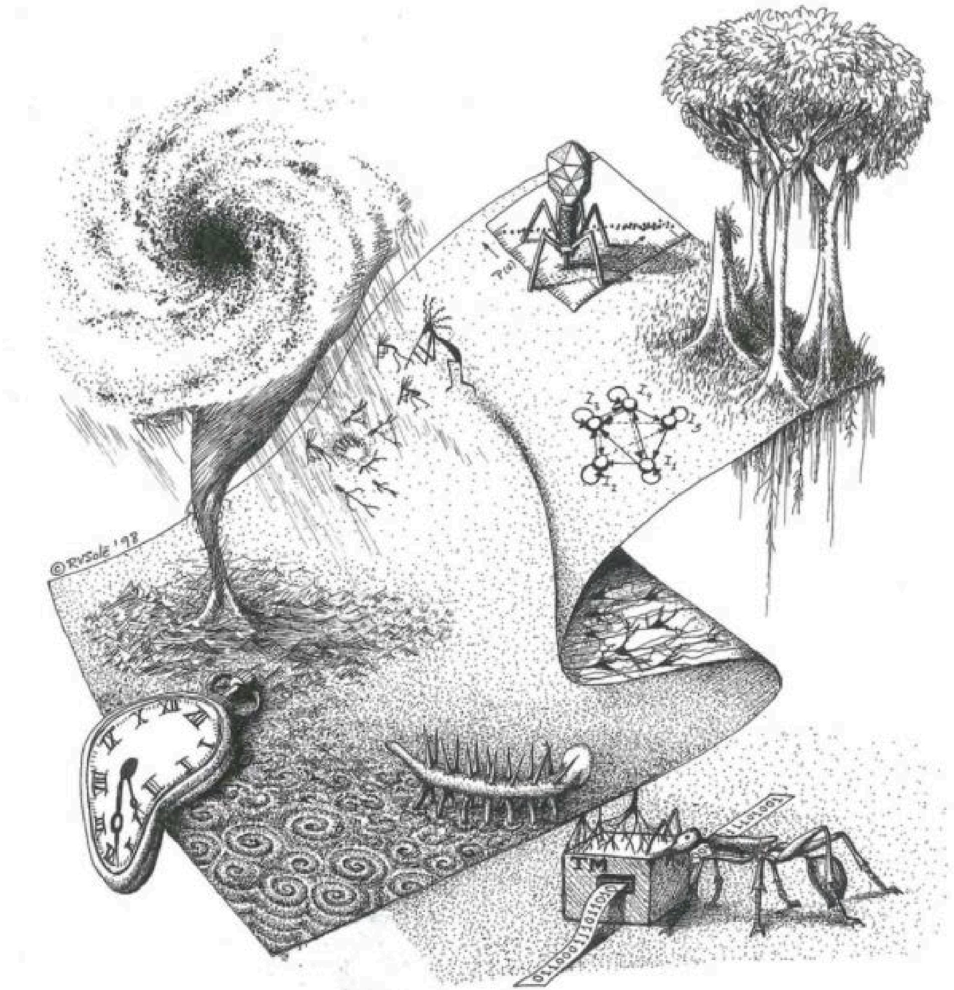


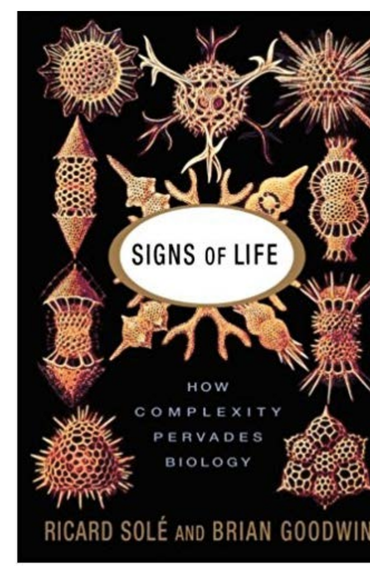
THE UNIVERSITY OF SYDNEY

Introduction



Introduction

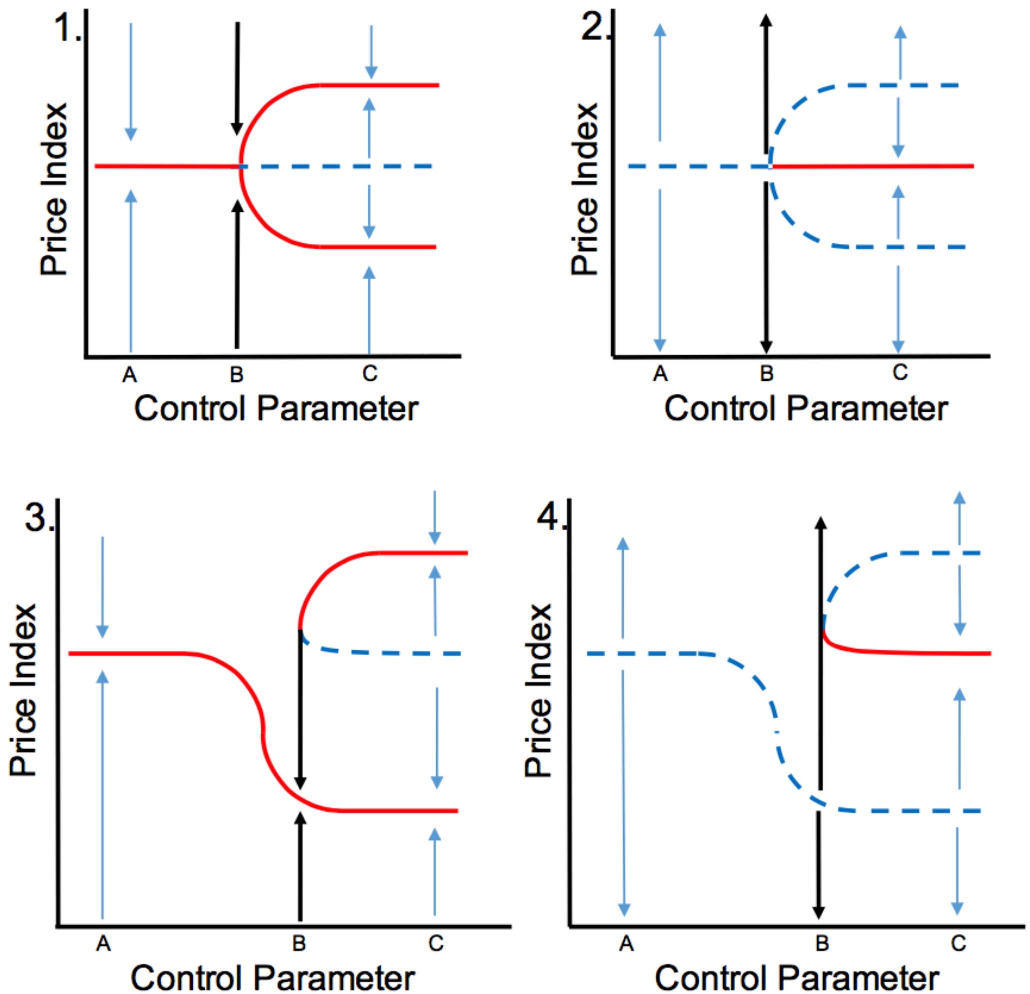




https://www.amazon.com/Signs-Life-Complexity-Pervades-Biology/dp/0465019285

Introduction

Rational explanations of markets?



Introduction

Discontinuous change in economic systems

Economic Theory:
Debreu's theory as an exemplar

Bifurcation Theory:
Thom's theory as an exemplar

Modern Synthesis:
Brock and Durlauf's Agent Based Models

Outline

1. Catastrophe Theory
2. Catastrophe Theory in Economics: Debreu and 'critical economies'
3. Game Theory and the Quantal Response Equilibrium
4. A Modern Approach

1. Catastrophe Theory

Ian Stewart's 1976 general discussion of Catastrophes (Euler's Arch)

$$V = x^4 - bx^2 + \alpha x \quad (+\ \text{const.})$$

Equilibrium positions of the arch are given by

$$0 = \frac{\partial V}{\partial x} = 4x^3 - 2bx + \alpha\ .$$

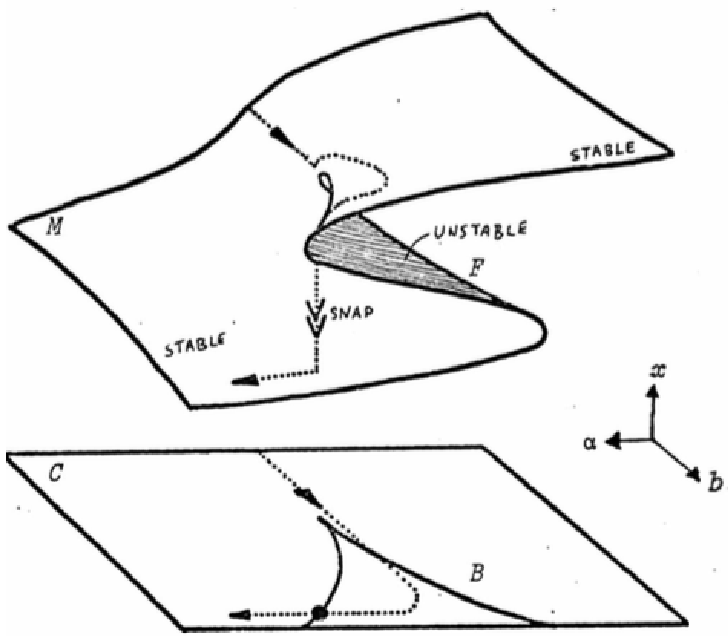


Figure 2.

The fold curve F occurs when

$$0 = \frac{\partial^2 V}{\partial x^2} = 12x^2 - 2b\ ,$$

$$27\alpha^2 = 8b^3$$

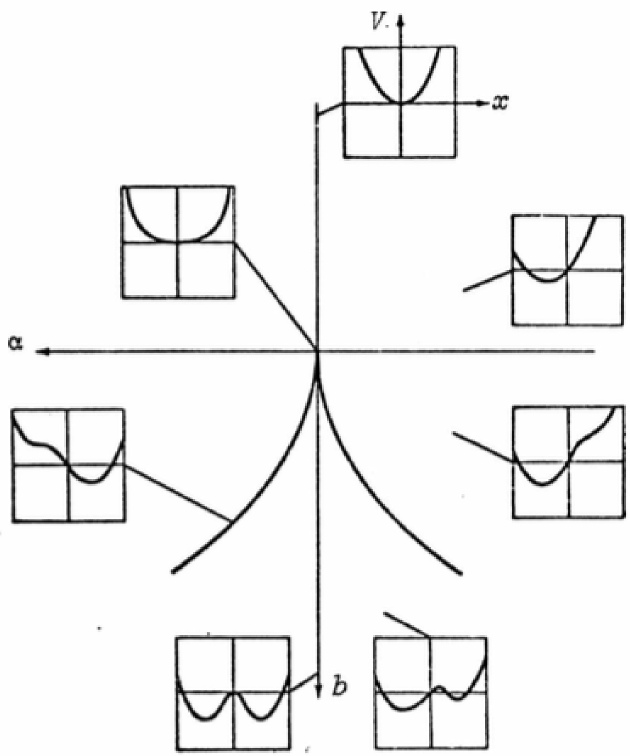
1. Catastrophe Theory

Ian Stewart's 1976 general discussion of Catastrophes (Euler's Arch)

$$V = x^4 - bx^2 + ax \quad (+\ \text{const.})$$

Equilibrium positions of the arch are given by

$$0 = \frac{\partial V}{\partial x} = 4x^3 - 2bx + a\ .$$



The fold curve F occurs when

$$0 = \frac{\partial^2 V}{\partial x^2} = 12x^2 - 2b\ ,$$

$$27a^2 = 8b^3$$

1. Catastrophe Theory

Stochastic Catastrophe Theory

$$dx_t = -\frac{\partial V(x_t|u_1,u_2)}{\partial x_t}dt$$

$$dx_t = -\frac{\partial V_\sigma(x_t|\mathbf{u})}{\partial x_t}dt + \sigma(x_t)dW_t$$

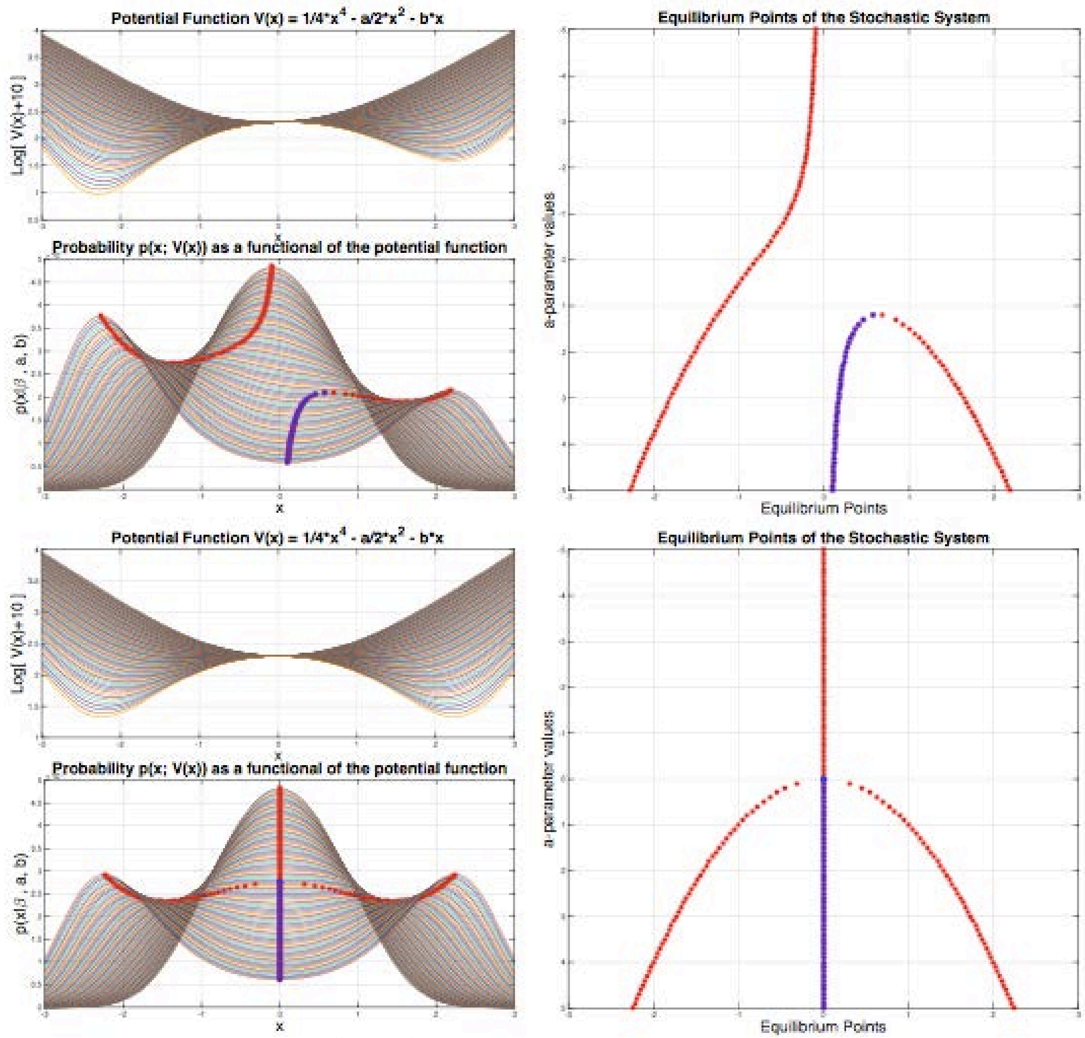
$$-\frac{\partial V(x_t|u_1,u_2)}{\partial x_t}\bigg|_x = -x^3 + u_1 x + u_2 = 0 \quad (\equiv \mu(x\ )\ \text{for brevity})$$

$$p(x|\mathbf{u},\sigma(x)) = \mathcal{Z}\exp\left(2\int_a^{x_t}\frac{\mu(z) - 0.5(d_z\sigma(z)^2)}{\sigma(z)^2}dz\right) \quad \left[d_z\sigma(z)^2 = \frac{d}{dz}\sigma(z)^2\right]$$

$$p(x|\mathbf{u},\xi) = \mathcal{Z}\exp\left(\xi\int_a^{x_t}\mu(z)dz\right) \quad \left[\ 2\sigma(z)^{-2} = \xi\ \right]$$

1. Catastrophe Theory

Stochastic Catastrophe Theory



1. Catastrophe Theory

Stochastic Catastrophe Theory

STOCHASTIC CATASTROPHE MODELS AND MULTIMODAL DISTRIBUTIONS[1]

by Loren Cobb

University of South Florida, Tampa

Nonlinear models such as have been appearing in the applied catastrophe theory literature are almost universally deterministic, as opposed to stochastic (probabilistic). The purpose of this article is to show how to convert a deterministic catastrophe model into a stochastic model with the aid of several reasonable assumptions, and how to calculate explicitly the resulting multimodal equilibrium probability density. Examples of such models from epidemiology, psychology, sociology, and demography are presented. Lastly, a new statistical technique is presented, with which the parameters of empirical multimodal frequency distributions may be estimated.

1. Catastrophe Theory

Stochastic Catastrophe Theory

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SCIENCE DIRECT

Physica D 211 (2005) 263–276

PHYSICA D

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Transformation invariant stochastic catastrophe theory☆

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Abstract

Catastrophe theory is a mathematical formalism for modeling nonlinear systems whose discontinuous behavior is determined by smooth changes in a small number of driving parameters. Fitting a catastrophe model to noisy data constitutes a serious challenge, however, because catastrophe theory was formulated specifically for deterministic systems. Loren Cobb addressed this challenge by developing a stochastic counterpart of catastrophe theory (SCT) based on Itô stochastic differential equations. In SCT, the stable and unstable equilibrium states of the system correspond to the modes and the antimodes of the empirical probability density function, respectively. Unfortunately, SCT is not invariant under smooth and invertible transformations of variables—this is an important limitation, since invariance to diffeomorphic transformations is essential in deterministic catastrophe theory. From the Itô transformation rules we derive a generalized version of SCT that does remain invariant under transformation and can include Cobb's SCT as a special case. We show that an invariant function is obtained by multiplying the probability density function with the diffusion function of the stochastic process. This invariant function can be estimated by a straightforward time series analysis based on level crossings. We illustrate the invariance problem and its solution with two applications.

PACS: 02.30.Oz; 02.50.Ey; 05.45.–a; 05.70.Fh

1. Catastrophe Theory

Stochastic Catastrophe Theory

This split between the original Thom formulation and the more generalized Arnol'd formulation also shows up in regard to the question of whether or not the systems must have a potential function, for which there is a necessary symmetry condition that all cross-partial derivatives must be equal. Again, the broader singularity theory does not require this, and forms that resemble the elementary catastrophes can appear within this theory even while not fulfilling the stricter assumption about the existence of a potential function. This would become a central issue in the later controversy over catastrophe theory. [1]

In Sandholm [8], potential games are defined using standard concepts from multivariable calculus. A population game F is called a potential game if there is a scalar-valued function f, the potential function, whose gradient always equals the vector of payoffs; put differently, the payoff to strategy i must always be given by the ith partial derivative of f. By standard results, F is a potential game in this sense if and only if its derivative matrices $DF(x)$ are symmetric, so that corresponding cross partial derivatives of F are equal. [2]

Quoted from: 1. *J.B. Rosser Jr. / Journal of Economic Dynamics & Control 31 (2007) 3255–3280*
2. *W.H. Sandholm / Journal of Economic Theory 144 (2009) 1710–1725*

1. Catastrophe Theory

Stochastic Catastrophe Theory: Evolutionary Game Theory

1. *Linear fitness.* We wish to include interactions between the types. The simplest possibility consists in considering replicator equations with linear fitness. With an interaction matrix $A = (a_{ij})_{i,j \in I}$, we consider the equation

$$\dot{p}_i = p_i \left(\sum_j a_{ij} p_j - \sum_k p_k \sum_j a_{kj} p_j \right). \tag{6.84}$$

While a constant fitness function could always be represented as a gradient field, in the linear case, by (2.38), we need the following condition:

$$a_{ij} + a_{jk} + a_{ki} = a_{ik} + a_{kj} + a_{ji}. \tag{6.85}$$

In particular, this is satisfied if the matrix A is symmetric, and a potential function then is

$$V(p) = \frac{1}{2} \sum_{ij} a_{ij} \, p_i p_j.$$

Replicator equation:
p_i = probability of state i

Quoted from: Nihat Ay et al *Information Geometry* (2017, pg. 331)

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Stochastic Catastrophe Theory: Evolutionary Game Theory

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$$-\left.\frac{\partial V(x_t|u_1,u_2)}{\partial x_t}\right|_x \quad = \quad -x^3 + u_1 x + u_2$$

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2. Catastrophe Theory in Economics

2. Catastrophe Theory in Economics

ECONOMETRICA

VOLUME 22 July, 1954 NUMBER 3

EXISTENCE OF AN EQUILIBRIUM FOR A COMPETITIVE ECONOMY

BY KENNETH J. ARROW AND GERARD DEBREU[1]

The assumptions made below are, in several respects, weaker and closer to economic reality than A. Wald's [23]. Unlike his models, ours presents an integrated system of production and consumption which takes account of the circular flow of income. The proof of existence is also simpler than his. Neither the uniqueness nor the stability of the competitive solution is investigated in this paper. The latter study would require specification of the dynamics of a competitive market as well as the definition of equilibrium.

2. Catastrophe Theory in Economics

Debreu and Singularity Theory: Econometrica article (1970)

Debreu stated: “A mathematical model which attempts to explain economic equilibrium must have a nonempty set of solutions ...” eventually concluding: “Our main result asserts that, under assumptions we will shortly make explicit ... every economy has a finite set of equilibria.”

Debreu knew of Thom’s work, but argued that so called ‘critical economies’ occurred negligibly often, i.e. the subset of all economic states that are exactly critical has measure zero.

While this is true, it misses the point of an economy transitioning through a critical state.

2. Catastrophe Theory in Economics

Debreu and Singularity Theory: Nobel prize (1983)

ECONOMIC THEORY IN THE MATHEMATICAL MODE

Nobel Memorial lecture, 8 December, 1983

by
GERARD DEBREU

"The explanation of equilibrium given by a model of the economy would be complete if the equilibrium were unique, and the search for satisfactory conditions guaranteeing uniqueness has been actively pursued … However, the strength of the conditions that were proposed made it clear by the late sixties that **global uniqueness was too demanding** a requirement and that one would have to be satisfied with local uniqueness. Actually, that property of an economy could not be guaranteed even under strong assumptions about the characteristics of the economic agents. But one can prove, as I did in 1970, that, under suitable conditions, in the set of all economies, **the set of economies that do not have a set of locally unique equilibria is negligible**."

2. Catastrophe Theory in Economics

Debreu and Singularity Theory: Nobel prize (1983)

ECONOMIC THEORY IN THE MATHEMATICAL MODE

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The i^{th} consumer is characterized by his demand function f_i … [T]he economy is described by the list $\mathbf{e} = (e_1, ..., e_m)$ of the m endowment-vectors … [and] the price vector p

$$F(p, e) = \sum_{i=1}^{m} \left[f_i\,(p, p \cdot e_i) - e_i\right].$$

For which equilibrium means the "excess demand" is zero: $F(p,e) = 0$

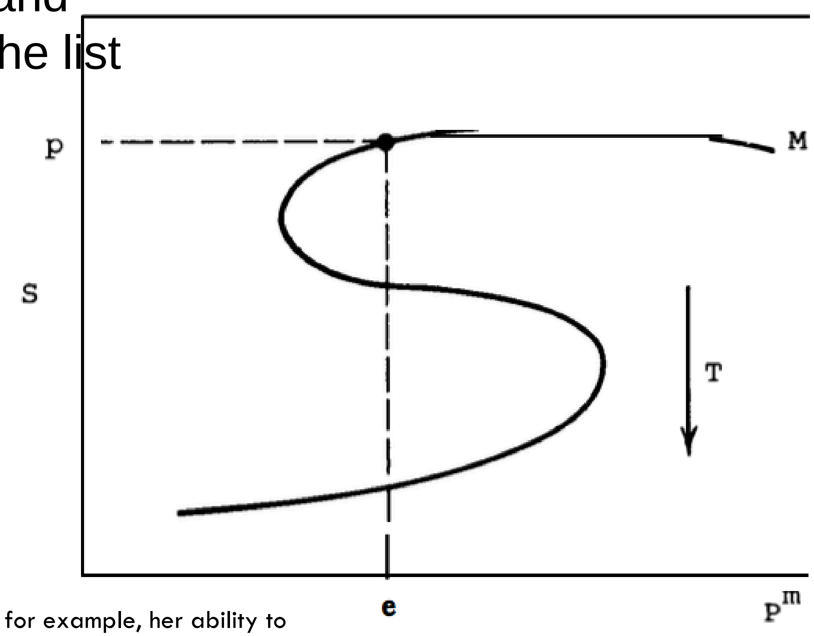

Figure 5

The endowment of an agent is the set of all goods she inherits: for example, her ability to labor, the apples on the trees in her backyard and so on.

J.D.Farmer: *The Virtues and Vices of Equilibrium and the Future of Financial Economics* (2008)

2. Catastrophe Theory in Economics

Debreu and Singularity Theory: Nobel prize (1983)

**ECONOMIC THEORY IN
THE MATHEMATICAL MODE**

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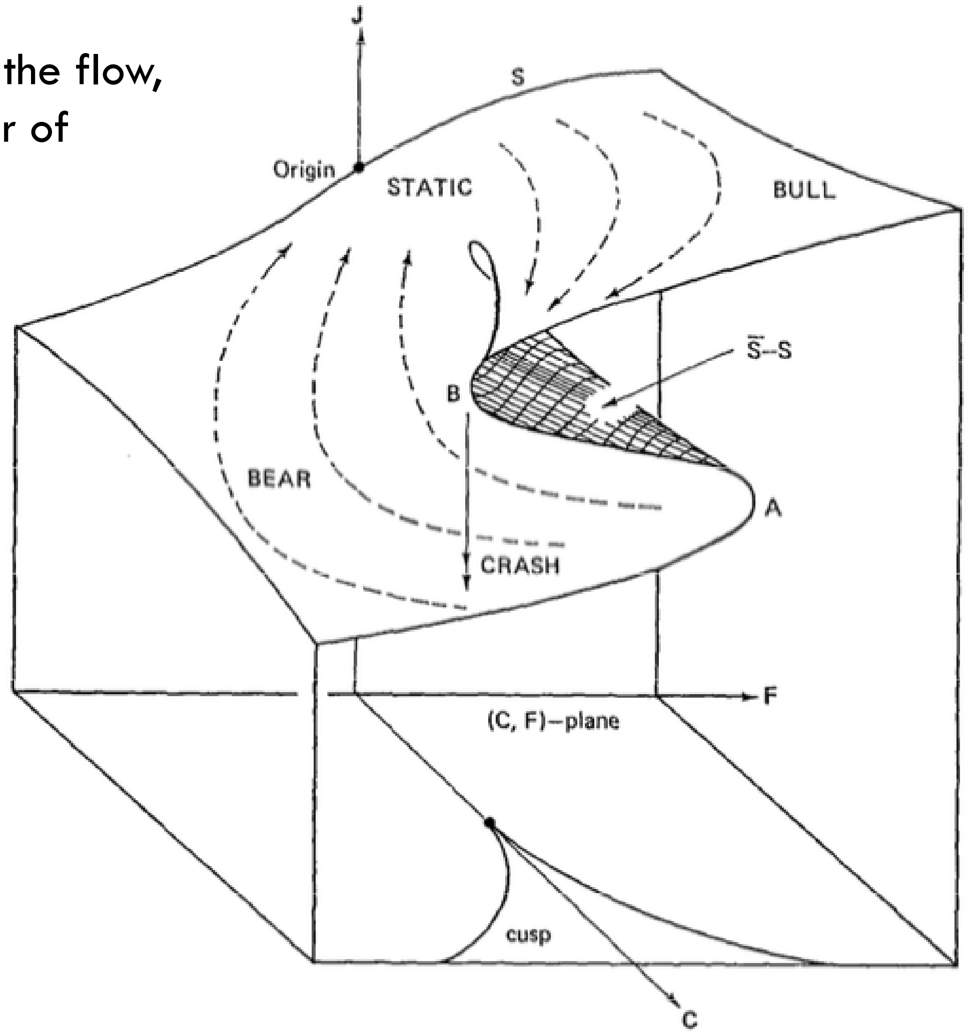
"Now let *T* be the projection from *M* into P^m and define a critical economy *e* as an economy such that it is the projection of a point (*e*,*p*) of *M* where the Jacobian of T is singular, geometrically where the tangent linear manifold of dimension *l* x *m* does not project onto P^m. By Sard's theorem **the set of critical economies is closed and of Lebesgue measure zero**. A regular economy, outside the negligible critical set, not only has a discrete set of equilibria; it also has a neighborhood in which the set of equilibria varies continuously as a function of the parameters defining the economy."

2. Catastrophe Theory in Economics

Financial Markets: Zeeman (1974 and 1976)

To build up the qualitative picture of the flow, we shall take as hypotheses a number of observed qualitative features of stock exchanges and currencies, and translate each feature into mathematics.



2. Catastrophe Theory in Economics

Financial Markets: Zeeman (1974 and 1976)

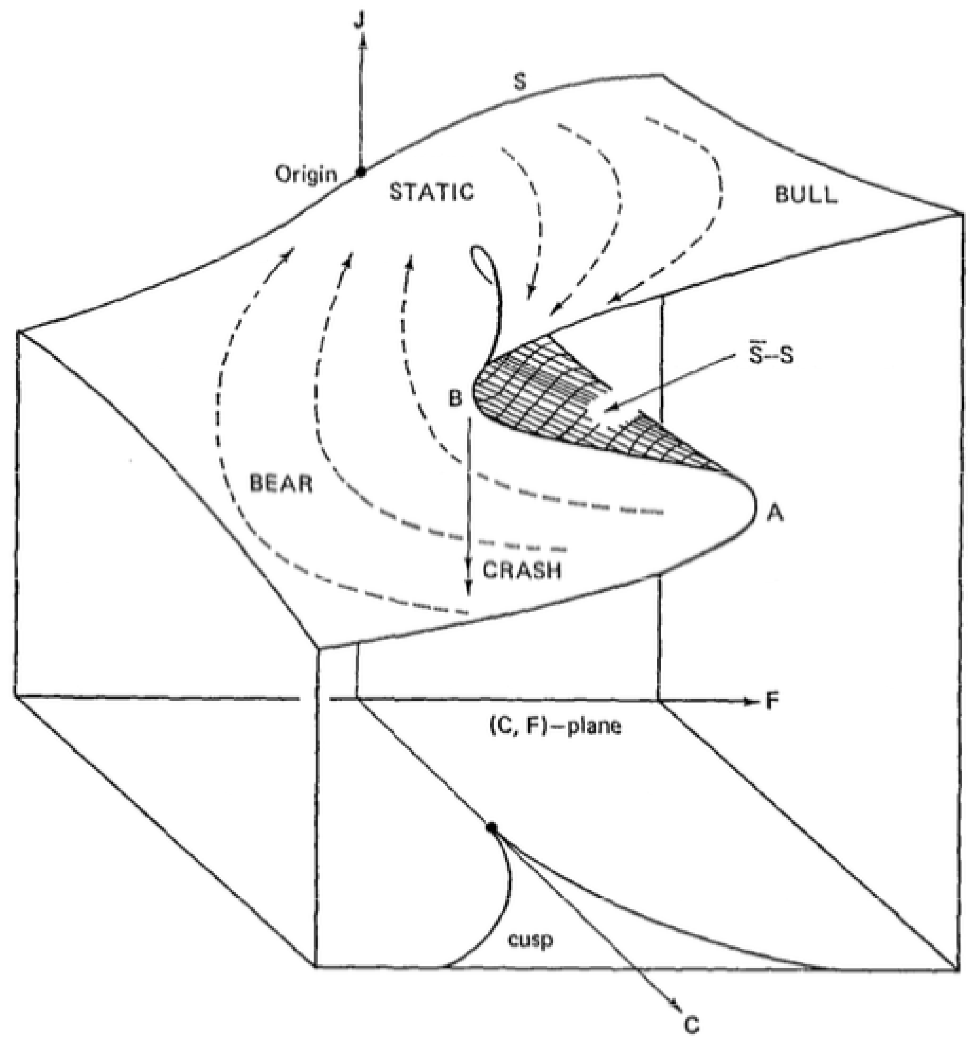
Two types of 'agents':
Fundamentalists
Chartists

Proportion of market that are chartists: C

Fundamentalist excess demand: F

Market index: J

$$J^3 - (C - C_0)J - F = 0$$



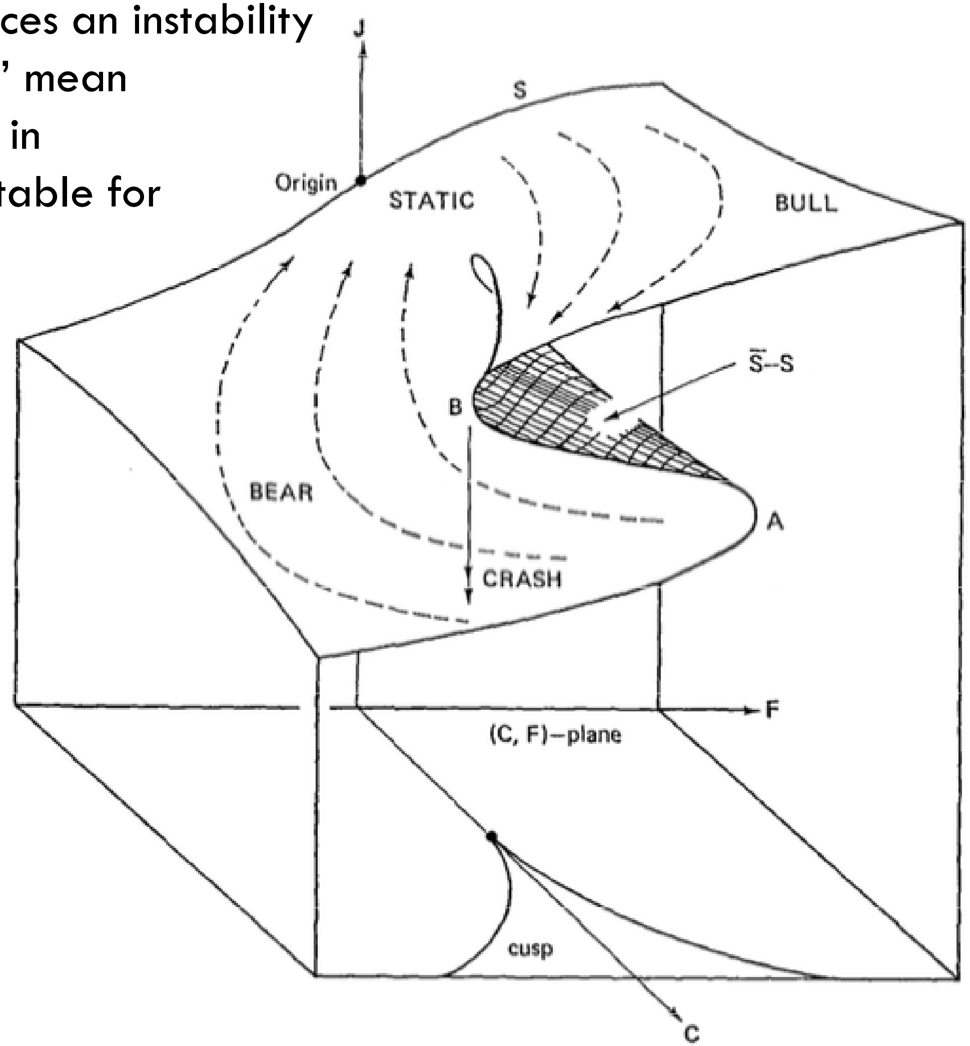
2. Catastrophe Theory in Economics

Financial Markets: Zeeman (1974 and 1976)

Hypothesis 3. If C is large this introduces an instability into the market. What does 'instability' mean mathematically? …we are postulating in Hypothesis 3 that it is dynamically unstable for the index to remain constant. Any slight perturbation of the index up or down (by external forces) will at once be amplified by the chartists.

…

The critical consequence of **Hypothesis 3** is that for large C and small F *the function* $J(C, F)$ *is 2-valued*, and so *the attractor surface* S *is 2-sheeted.*



2. Catastrophe Theory in Economics

The rise and fall of catastrophe theory applications in economics: Was the baby thrown out with the bathwater?

J. Barkley Rosser Jr.

The most important criticisms of catastrophe theory applications in general were by Zahler and Sussman (1977), Sussman and Zahler (1978a, b),[9] and Kolata (1977). Responses appeared in *Science* and *Nature* in 1977, with a more vigorous and extended set of defenses appearing in *Behavioral Science* (Oliva and Capdeville, 1980; Guastello, 1981),[10] with the first of these being the source of the line that 'the baby was thrown out with the bathwater.' More balanced overviews came from mathematicians (Guckenheimer, 1978; Arnol'd, 1992).

2. Catastrophe Theory in Economics

The rise and fall of catastrophe theory applications in economics: Was the baby thrown out with the bathwater?

J. Barkley Rosser Jr.

The critics succeeded in pointing out some dirty bathwater.[11] The most salient points include: (1) excessive reliance on qualitative methods, (2) inappropriate quantization in some applications, and (3) the use of excessively restrictive or narrow mathematical assumptions. The third point in turn has at least three sub-points: (a) the necessity for a potential function to exist for it to be properly used, (b) that the necessary use of gradient dynamics does not allow the use of time as a control variable as was often done in many applications, and (c) that the set of elementary catastrophes is only a limited subset of the possible range of bifurcations and catastrophes. These arguments relate to applications of catastrophe theory in general rather than to economics specifically.

2. Catastrophe Theory in Economics

Financial Markets: Plerou et al.

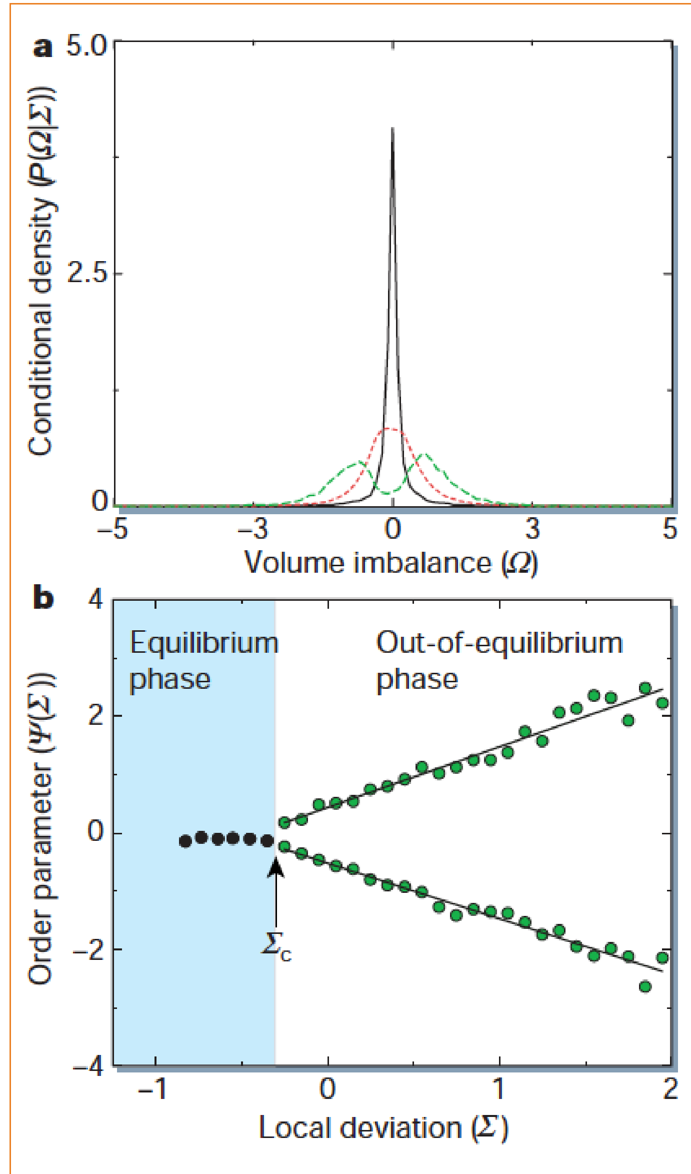
Q_B = buyer initiated transactions
Q_S = seller initiated transactions

$$\Omega(t) \equiv Q_{\mathrm{B}} - Q_{\mathrm{S}} = \sum_{i=1}^{N} q_i a_i$$

$$\Sigma(t) \equiv \langle | q_i a_i - \langle q_i a_i \rangle | \rangle$$

The order paramet $\Psi = \Psi(\Sigma)$ is given by the values of the maxi $\Omega_\pm$ of $P(\Omega)$ of .

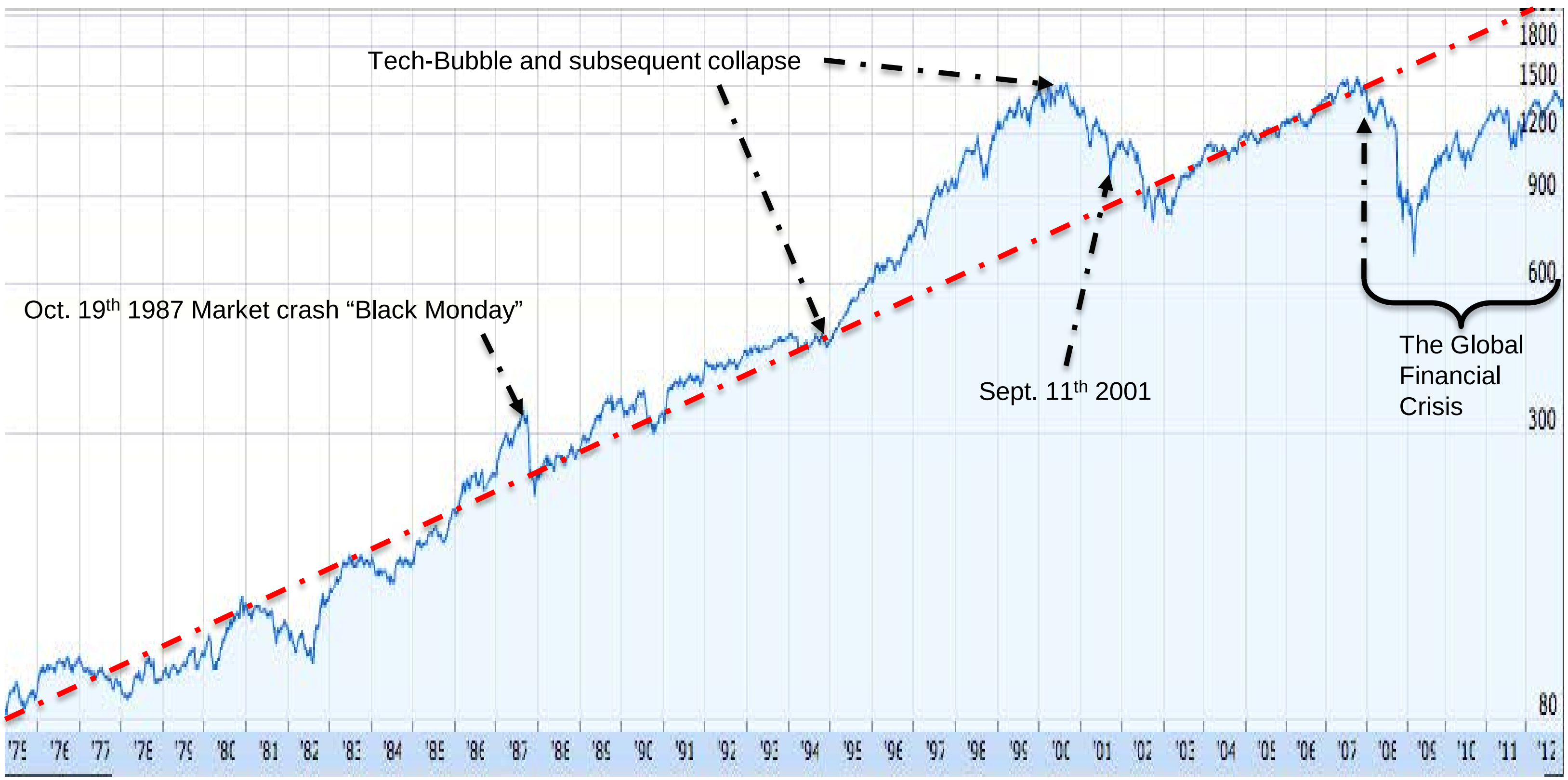
$$\Psi(\Sigma) = \begin{cases} 0 & [\Sigma < \Sigma_{\mathrm{c}}] \\ \Sigma - \Sigma_{\mathrm{c}} & [\Sigma \gg \Sigma_{\mathrm{c}}] \end{cases}$$



Plerou, Gopikrishnan, Stanley: "Two-phase behaviour of financial markets", *Nature* (2003)

2. Catastrophe Theory in Economics

Financial Markets: Data for the S&P500 (log-linear)



2. Catastrophe Theory in Economics

Financial Markets: Data for the S&P500 (log-linear)

2. Catastrophe Theory in Economics

Housing Markets: US Housing Market Collapse

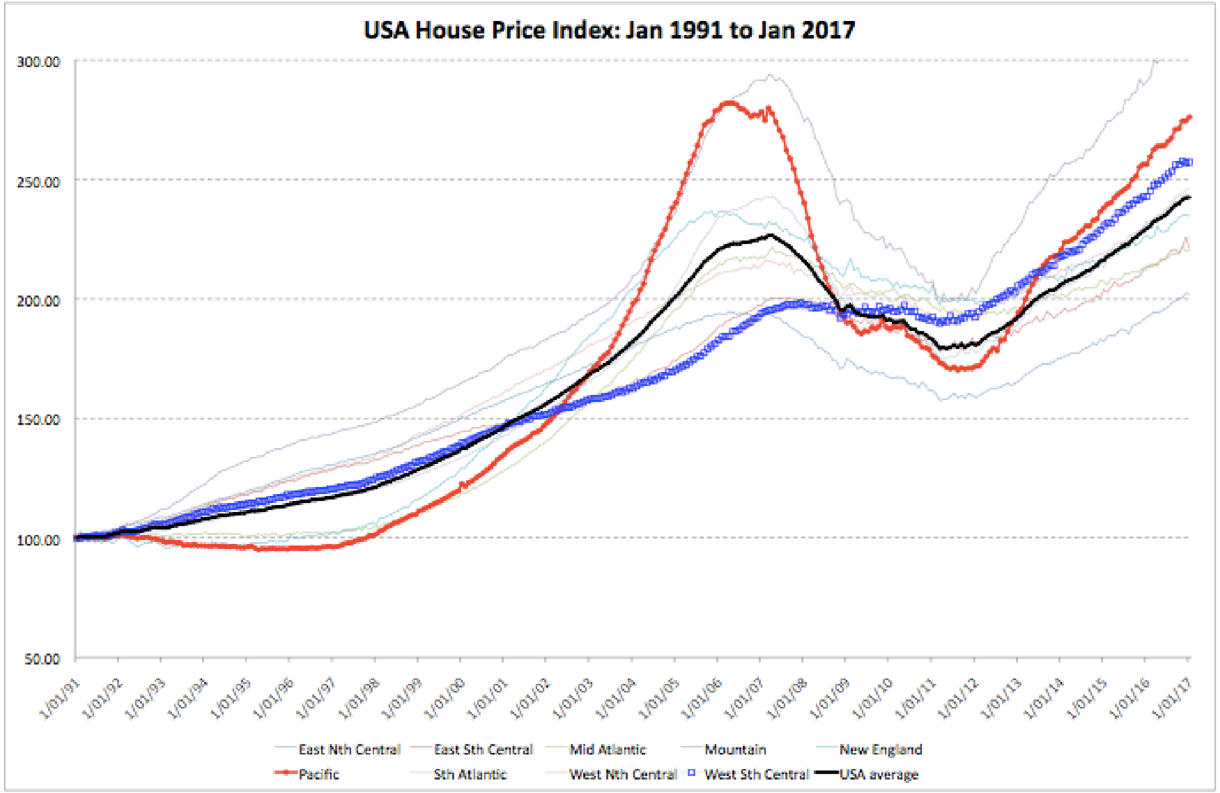


Figure 2: The impact of the boom and bust in the US housing market was regionally heterogeneous, seasonally adjusted monthly house price indices for the nine Census Bureau Divisions of the United States, indices set to 100 on January 1^{st} 1991.

M.Harre: *Agent Based Models of Market Price Dynamics*, in preparation

2. Catastrophe Theory in Economics

Housing Markets: US Housing Market Collapse

Bolt, Wilko; Demertzis, Maria; Diks, Cees; Hommes, Cars; van der Leij, Marco

Working Paper

Identifying Booms and Busts in House Prices under Heterogeneous Expectations

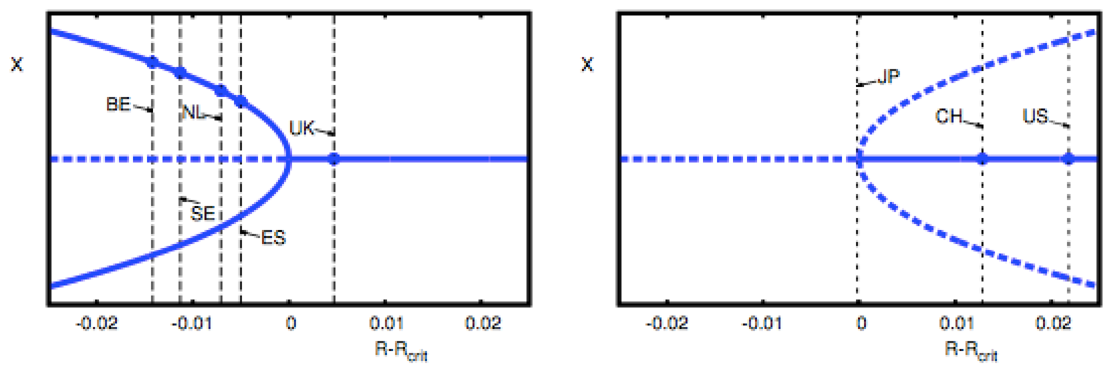


Figure 12: Pitchfork bifurcation w.r.t. the policy parameter R. Bold curves denote stable steady states; dotted curves are unstable steady states. **Left Panel:** supercritical pitchfork bifurcation with stable fundamental steady state for $R > R_{crit}$ and unstable fundamental steady state surrounded by two stable non-fundamental steady states for $R < R_{crit}$. **Right Panel:** subcritical pitchfork bifurcation with stable fundamental steady state surrounded by a corridor of stability bounded by two unstable non-fundamental steady states for $R > R_{crit}$ and (globally) unstable fundamental steady state with exploding dynamics for $R < R_{crit}$. The dots represent the estimated values $R - R_{crit}$ for the eight countries.

2. Catastrophe Theory in Economics

Housing Markets: US Housing Market Collapse

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Journal of Economic Dynamics and Control

Volume 69, August 2016, Pages 68-88

Can a stochastic cusp catastrophe model explain housing market crashes?

Cees Diks, Juanxi Wang [1]

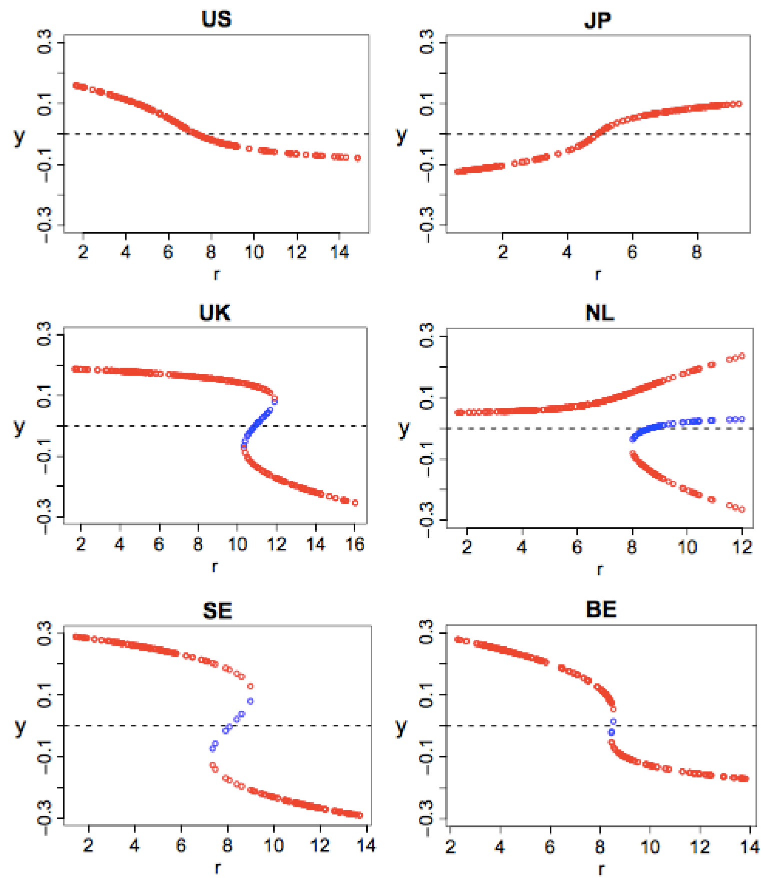


Fig. 10. Bifurcations showing the predicted equilibria as a function of the interest rate r for the different countries. Red scatter represents the stable equilibrium (up or bottom sheet). Blue scatter represents the unstable equilibrium (middle sheet).

3. The Quantal Response Equilibrium

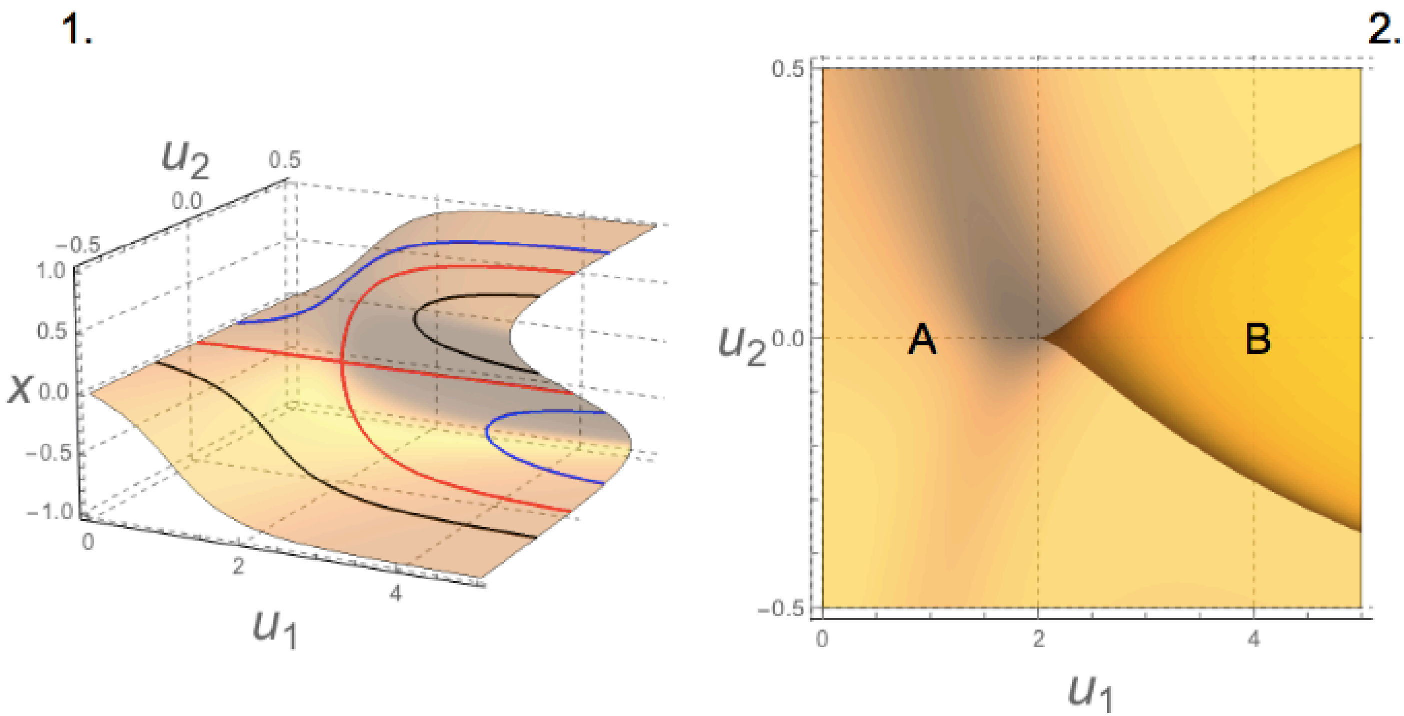


Figure 3: Plot 1.: The *Cusp catastrophe* with three contours shown for fixed u_2 but variable u_1 control parameters. A Pitchfork bifurcation is shown in red and two fold bifurcations are shown in black and blue. Plot 2.: Two distinct regions A and B can be discerned in the projection of the equilibrium surface onto the control plane $\{u_1, u_2\}$, region A has one equilibrium point and region B has three equilibrium points.

3. The Quantal Response Equilibrium

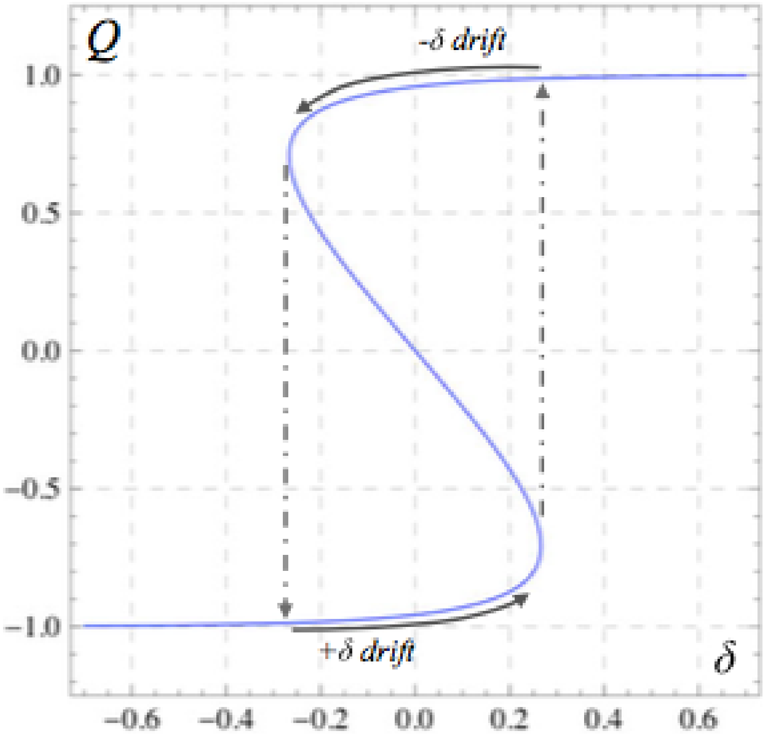
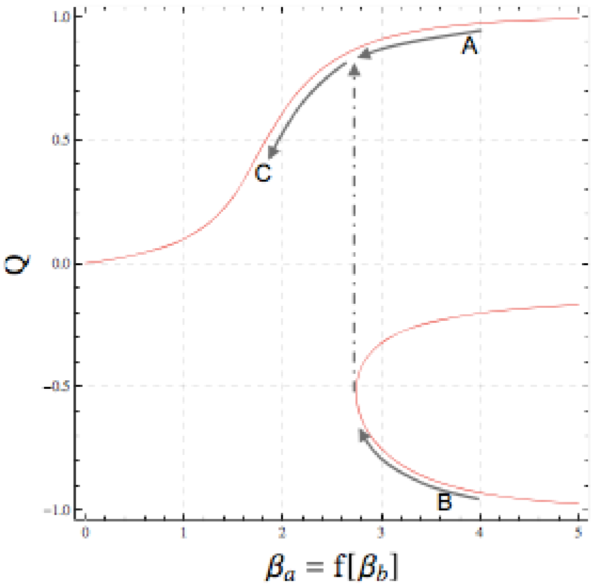
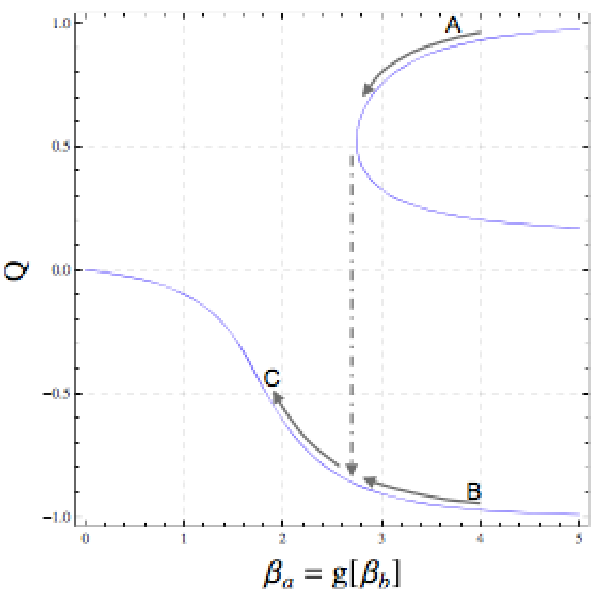


Fig. 1. Hysteresis and the cyclical collapse of a system that is drifting across an equilibrium surface. Q can be thought of as a behavioural outcome, dictated by the system structure and its parameters.





3. The Quantal Response Equilibrium

A game is a function

$$g : S_1 \times \ldots \times S_n \rightarrow \mathbb{R}^n,$$

where $g(s_1, \ldots, s_n) = (g_1(s_1, \ldots, s_n), \ldots g_n(s_1, \ldots, s_n))$, and $g_i(s_1, \ldots, s_n)$ is player i's payoff when strategies $(s_1, \ldots, s_n)$ are played

Dasgupta and Maskin: Debreu's social equilibrium existence theorem *PNAS* (2015)

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$$\text{player } i\text{'s expected payoff} =$$

$$\sum_{s_1 \in S_1} \cdots \sum_{s_n \in S_n} g_i(s_1, \ldots, s_n)\, p_1(s_1) \ldots p_n(s_n)$$

$$= g_i(p_1, \ldots, p_n) = g_i(p_i, p_{-i}).$$

Dasgupta and Maskin: Debreu's social equilibrium existence theorem *PNAS* (2015)

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$$\text{player } i\text{'s expected payoff} = \sum_{s_1 \in S_1} \cdots \sum_{s_n \in S_n} g_i(s_1, \ldots, s_n)\, p_1(s_1) \ldots p_n(s_n)$$

$$= g_i(p_1, \ldots, p_n) = g_i(p_i, p_{-i}).$$

Nash proved that there exists an n-tuple $(p_1^*, \ldots, p_n^*)$ such that

$$g_i(p_1^*, \ldots, p_n^*) \geq g_i(p_i, p_{-i}^*) \quad \text{for all } i \text{ and } p_i.$$

Dasgupta and Maskin: Debreu's social equilibrium existence theorem *PNAS* (2015)

3. The Quantal Response Equilibrium

Nash Equilibrium:

$$u^a_{i,j} = \begin{bmatrix} u^a_{1,1} & u^a_{1,2} \\ u^a_{2,1} & u^a_{2,2} \end{bmatrix}, \quad u^b_{i,j} = \begin{bmatrix} u^b_{1,1} & u^b_{1,2} \\ u^b_{2,1} & u^b_{2,2} \end{bmatrix}$$

$$E(u^a) = \sum_{i,j} p_i q_j u^a_{i,j}, \quad E(u^b) = \sum_{i,j} p_i q_j u^b_{i,j}.$$

p_i in [0,1] and q_j in [0,1]
$p_1 + p_2 = 1$, $q_1 + q_2 = 1$

$$\left.\begin{aligned} p^*_i &= \mathrm{argmax}_{p_i} \sum_{i,j} p_i q^*_j u^a_{i,j} \; \forall i \\ q^*_j &= \mathrm{argmax}_{q_j} \sum_{i,j} p^*_i q_j u^b_{i,j} \forall j. \end{aligned}\right\}$$

Nash Equilibrium in p and q: <u>not</u> unique

3. The Quantal Response Equilibrium

Quantal Response Equilibrium:

$$p_i^* = \max_{p_i} S(p_i) = \max_{p_i} \left(-\sum_i p_i \ln(p_i) \right)$$

subject to the constraints:

$$p_i \geq 0 \, \forall \, i, \;\; \sum_i p_i \; = \; 1, \;\; \sum_{i,j} p_i q_j u^a_{i,j} \; = \; E\left(u^a\right).$$

D. Wolpert M. Harre (2013)

3. The Quantal Response Equilibrium

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subject to the constraints:

$$p_i \geq 0 \; \forall \, i, \;\; \sum_i p_i \; = \; 1, \;\; \sum_{i,j} p_i q_j u^a_{i,j} \; = \; E\left(u^a\right).$$

$$\mathcal{L}(q_i) = S(p_i) + \beta_a \sum_{i,j} p_i q_j u^a_{i,j} + \beta_0 \sum_i p_i,$$

$$\frac{\partial \mathcal{L}(p_i)}{\partial p_i} = -\ln(p_i^*) + \beta_a \sum_j q_j u^a_{i,j} + \beta_0 - 1 = 0,$$

$$p_i^* = \mathcal{Z}_a^{-1} \exp\left(\beta_a \sum_j q_j^* u^a_{i,j} \right),$$

3. The Quantal Response Equilibrium

$$Q_r = f(Q_c, \beta_r) = 1 - \frac{2e^{\beta_r \mathbb{E}(u_r|top)}}{e^{\beta_r \mathbb{E}(u_r|top)} + e^{\beta_r \mathbb{E}(u_r|bottom)}}$$

$$Q_c = g(Q_r, \beta_c) = 1 - \frac{2e^{\beta_c \mathbb{E}(u_c|left)}}{e^{\beta_c \mathbb{E}(u_c|left)} + e^{\beta_c \mathbb{E}(u_c|right)}}$$

Q_r in [-1,1]
and
Q_c in [-1,1]

$$Q_r = \tanh\left[2\beta_r\left(f_r(U_r) + f_{c,r}(U_r)Q_c\right)\right]$$

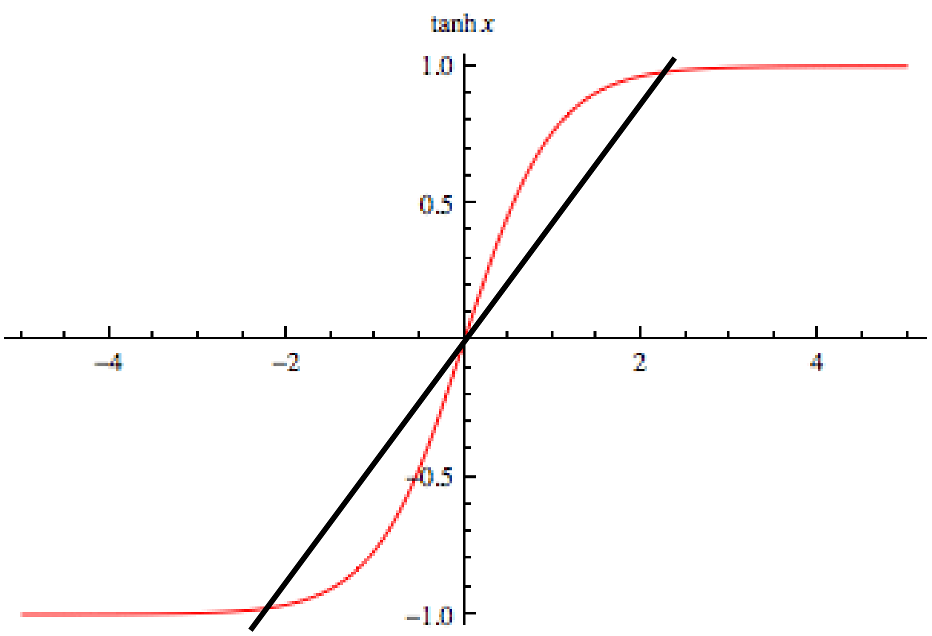
$$Q_c = \tanh\left[2\beta_c\left(f_c(U_c) + f_{c,r}(U_c)Q_r\right)\right]$$

$$G(Q_c, \beta_c, \beta_r) \triangleq Q_c - g(Q_r, \beta_c)$$

$$= Q_c - g(f(Q_c, \beta_r), \beta_c)$$

$$F(Q_r, \beta_r, \beta_c) \triangleq Q_r - f(Q_c, \beta_r)$$

$$= Q_r - f(g(Q_r, \beta_c), \beta_r)$$



3. The Quantal Response Equilibrium

$$0 = \frac{\partial Q_r}{\partial \beta_r} - \left(\frac{\partial f}{\partial \beta_r} + \frac{\partial f}{\partial g}\frac{\partial g}{\partial Q_r}\frac{\partial Q_r}{\partial \beta_r}\right)$$

$$\frac{\partial f}{\partial \beta_r} = \frac{\partial Q_r}{\partial \beta_r}\left(1 - \frac{\partial g}{\partial Q_r}\frac{\partial f}{\partial g}\right)$$

$$\frac{\partial Q_r}{\partial \beta_r} = \frac{\frac{\partial f}{\partial \beta_r}}{1 - \frac{\partial g}{\partial Q_r}\frac{\partial f}{\partial g}}$$

$$\frac{\partial Q_r}{\partial \beta_r} = \frac{f_2(Q_c, \beta_r)}{1 - g_1(Q_r, \beta_c) f_1(Q_c, \beta_r)} = \frac{f_2}{1 - g_1 f_1}$$

3. The Quantal Response Equilibrium

$$0 = \frac{\partial Q_r}{\partial \beta_r} - \left(\frac{\partial f}{\partial \beta_r} + \frac{\partial f}{\partial g}\frac{\partial g}{\partial Q_r}\frac{\partial Q_r}{\partial \beta_r}\right)$$

$$\frac{\partial f}{\partial \beta_r} = \frac{\partial Q_r}{\partial \beta_r}\left(1 - \frac{\partial g}{\partial Q_r}\frac{\partial f}{\partial g}\right)$$

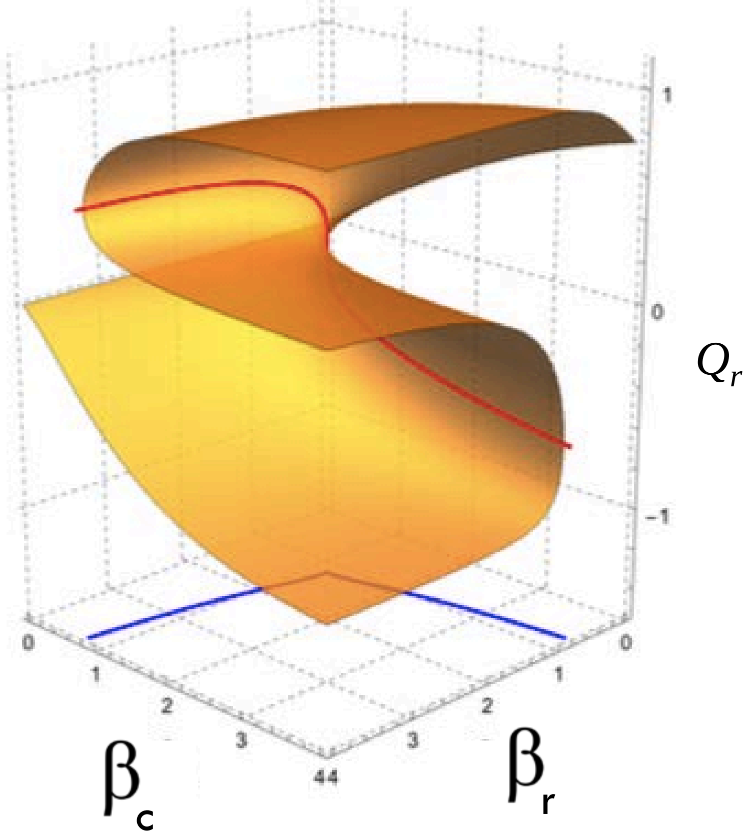
$$\frac{\partial Q_r}{\partial \beta_r} = \frac{\frac{\partial f}{\partial \beta_r}}{1 - \frac{\partial g}{\partial Q_r}\frac{\partial f}{\partial g}}$$

$$\frac{\partial Q_r}{\partial \beta_r} = \frac{f_2(Q_c, \beta_r)}{1 - g_1(Q_r, \beta_c) f_1(Q_c, \beta_r)} = \frac{f_2}{1 - g_1 f_1}$$

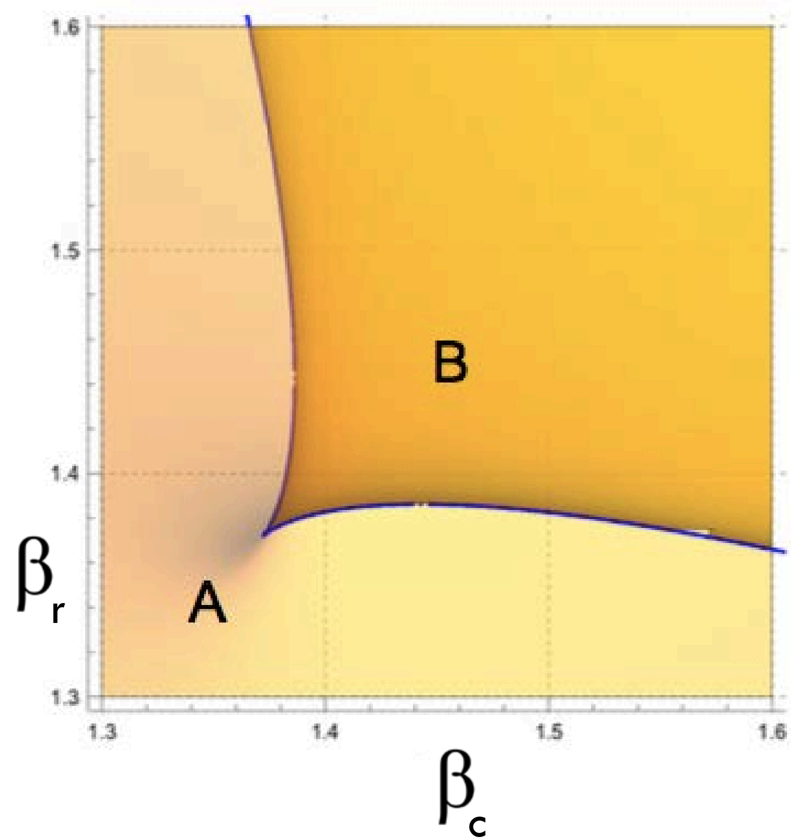
$$\left.\begin{aligned} \frac{\partial Q_r}{\partial \beta_r} &= \frac{f_2}{1 - f_1 g_1}, & \frac{\partial Q_r}{\partial \beta_c} &= \frac{f_1 g_2}{1 - f_1 g_1} \\ \frac{\partial Q_c}{\partial \beta_r} &= \frac{g_1 f_2}{1 - f_1 g_1}, & \frac{\partial Q_c}{\partial \beta_c} &= \frac{g_2}{1 - f_1 g_1} \end{aligned}\right\} \frac{1}{1 - f_1 g_1}\begin{pmatrix} f_2 & f_1 g_2 \\ g_1 f_2 & g_2 \end{pmatrix}$$

3. The Quantal Response Equilibrium

Equilibrium surface



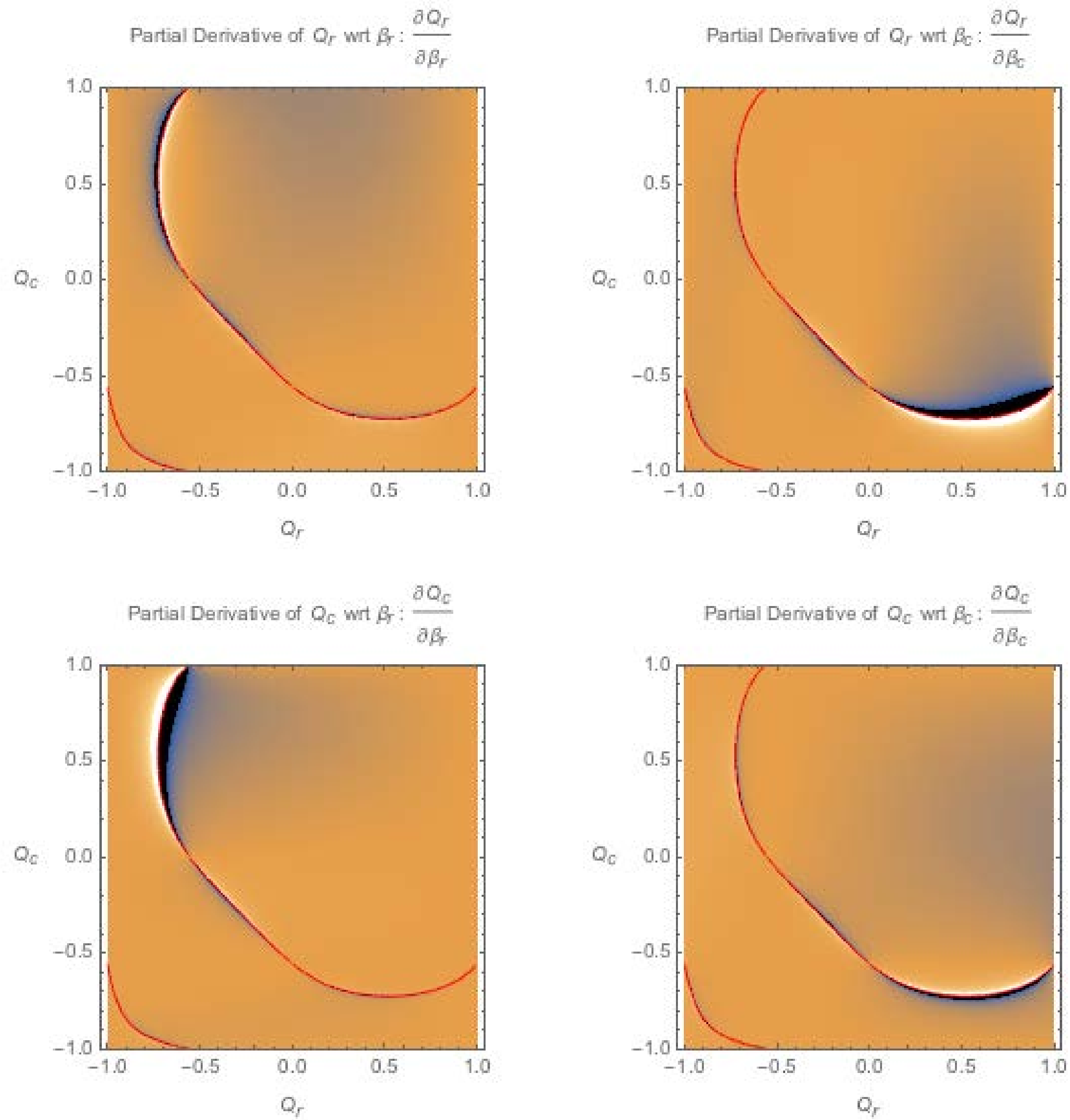
Bifurcation set of ‘critical economies’



Curves are values of β_i for which: $1 = f_1 g_1$

3. The Quantal Response Equilibrium

The bifurcation set in Q_r x Q_c partitions the possible states of a market/economy.

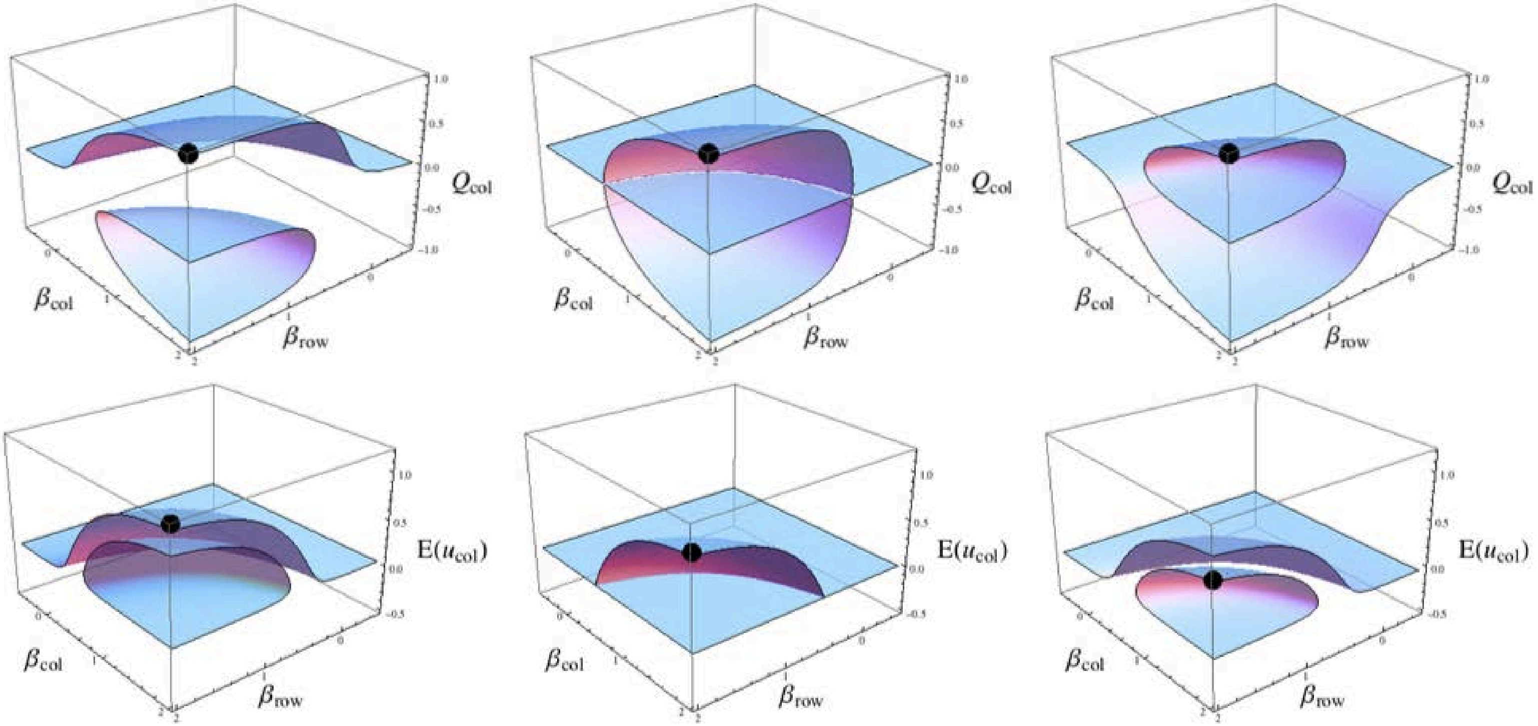


3. The Quantal Response Equilibrium

$$Q_r = \tanh\left[2\beta_r\left(f_r(U_r) + f_{c,r}(U_r)Q_c\right)\right]$$

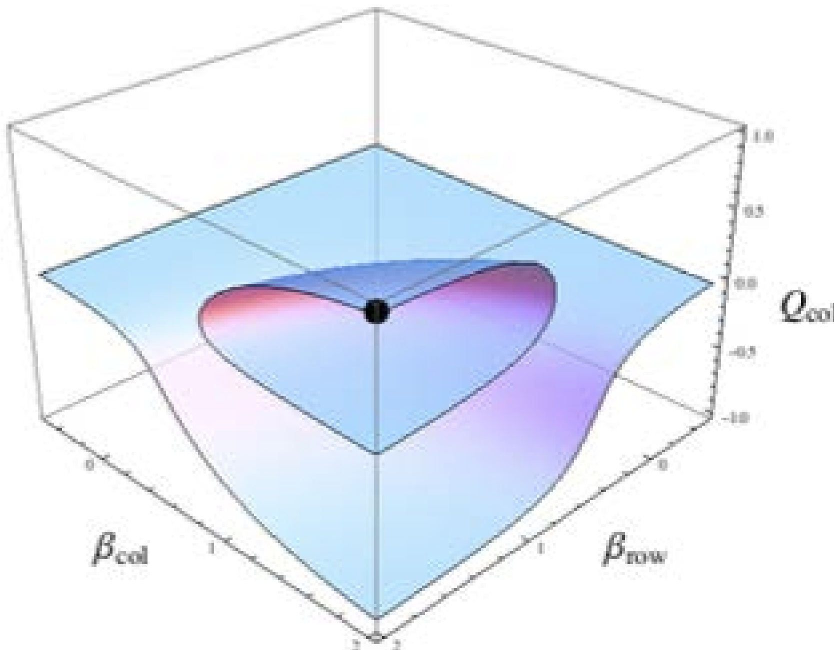
$$Q_c = \tanh\left[2\beta_c\left(f_c(U_c) + f_{c,r}(U_c)Q_r\right)\right]$$

Figure 4. Perturbed QRE solutions for $\delta_c = \delta_r \in \{0.2, 0, -0.2\}$ from left to right with a β pair $\beta_c = \beta_r = 2$, the equilibrium strategy is where the black dot is, see Equations (38)–(39).



3. The Quantal Response Equilibrium

$$Q_r = \tanh\left[2\beta_r\left(f_r(U_r) + f_{c,r}(U_r)Q_c\right)\right]$$
$$Q_c = \tanh\left[2\beta_c\left(f_c(U_c) + f_{c,r}(U_c)Q_r\right)\right]$$



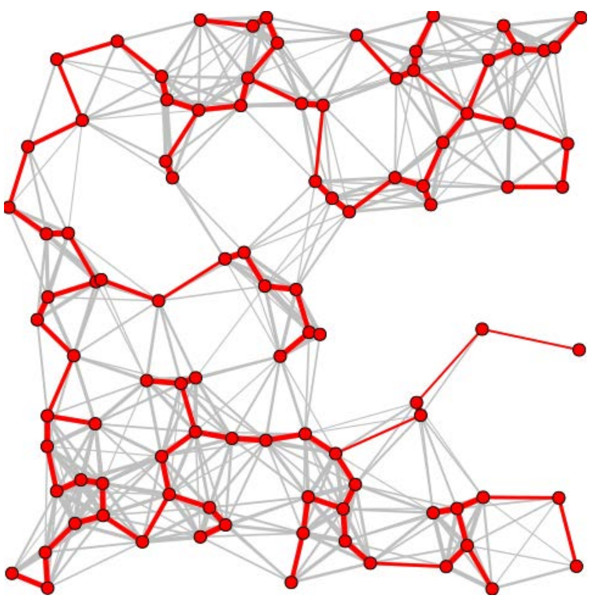
Variations in β_{row} results in an induced bifurcation in Q_{col}

4. A Modern Treatment

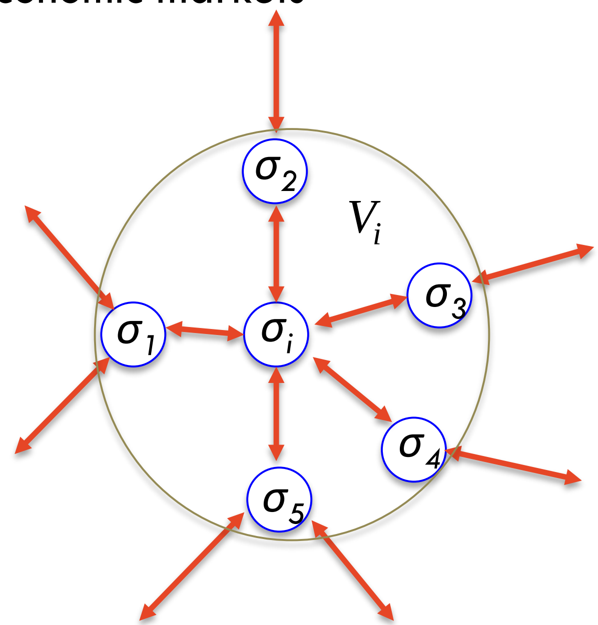
$$
\begin{aligned}
Q_r &= \tanh\left[2\beta_r\big(f_r(U_r) + f_{c,r}(U_r)Q_c\big)\right] \\
Q_c &= \tanh\left[2\beta_c\big(f_c(U_c) + f_{c,r}(U_c)Q_r\big)\right]
\end{aligned}
$$

$$
\langle\sigma_i\rangle = \tanh\left(\xi_i\Big(h_i + \sum_{j\in\mathcal{V}_i} J_{i,j}\langle\sigma_j^e\rangle\Big)\right)
$$

Brock and Durlauf
Heterogeneous Agent Models in Economics and Finance (2006)



Modelling for "neighborhood effects" in economic markets



4. A Modern Treatment

$$
\begin{aligned}
Q_r &= \tanh\left[2\beta_r\big(f_r(U_r) + f_{c,r}(U_r)Q_c\big)\right] \\
Q_c &= \tanh\left[2\beta_c\big(f_c(U_c) + f_{c,r}(U_c)Q_r\big)\right]
\end{aligned}
$$

$$
\langle\sigma_i\rangle = \tanh\Big(\xi_i\big(h_i + \sum_{j\in\mathcal{V}_i} J_{i,j}\langle\sigma_j^e\rangle\big)\Big)
$$

Brock and Durlauf
Heterogeneous Agent Models in Economics and Finance (2006)

"**endogenous effects,** wherein the propensity of an individual to behave in some way varies with the behaviour of the group … **exogenous (contextual)** effects, wherein the propensity of an individual to behave in some way varies with the exogenous characteristics of a group … **correlated** effects, wherein individuals in the same group tend to behave similarly because they have similar individual characteristics or face similar institutional environments"
Manski (1993, pg. 532)

4. A Modern Treatment

$$Q_r = \tanh\left[2\beta_r\left(f_r(U_r) + f_{c,r}(U_r)Q_c\right)\right]$$
$$Q_c = \tanh\left[2\beta_c\left(f_c(U_c) + f_{c,r}(U_c)Q_r\right)\right]$$

$$\langle\sigma_i\rangle = \tanh\left(\xi_i\left(h_i + \sum_{j\in\mathcal{V}_i} J_{i,j}\langle\sigma_j^e\rangle\right)\right)$$

Brock and Durlauf
Heterogeneous Agent Models in Economics and Finance (2006)

Chapter 54

INTERACTIONS-BASED MODELS

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Contents

Social Interactions

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Key Words

neighborhood effects, statistical mechanics, social interactions empirics, endogenous social effects, estimation and identification, social networks

Abstract

The study of social interactions has enriched both the domain of inquiry of economists and the way economists conceptualize individual decision making. The review aims to introduce the classes of models that accommodate estimation of social interactions and to examine the key areas where significant advances have been made in the identification of social effects. It surveys linear and nonlinear models and their applications, including results regarding partial identification. The review also examines conceptual and methodological links with the spatial econometrics and the social networks literatures. It considers empirical applications in the context of our methodological overview.

Questions

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