THE UNIVERSITY OF SYDNEY

Business School

Arithmetic

Mathematics Help Sheet

The University of Sydney Business School

Common Arithmetic Symbols

≠ ‘is not equal to’

≈ ‘is approximately equal to’

≡ ‘is identically equal to’

∞ ‘infinity’, which is a non-finite number

| x | ‘the absolute value’ of x, or the positive value of ‘x’. E.g. |-2| = 2.

> ‘greater than’

< ‘less than’

≥ ‘greater than or equal to’

≤ ‘less than or equal to’

=> ‘implies’. E.g., x = 5 => x + 3 = 8

/ ‘divide by’

Order of Operations

When you are solving a mathematical equation that contains a series of different operations, they must be calculated in a specific order. This order is **B**rackets, **O**rder, **D**ivision, **M**ultiplication, **A**ddition and then **S**ubtraction, or ‘**B O D M A S**’.

For example, to evaluate $7 + (6 \times 5^2 + 3)$,

1. Resolve the brackets first. Within the bracket, the ‘order’ is calculated first.
   $$7 + (6 \times 25 + 3)$$
2. Within the bracket, then perform multiplication, and then addition.
   $$7 + (150 + 3)$$
   $$7 + (153)$$
3. The last step is to perform the addition operation outside of the brackets.
   $$7 + 153 = 160$$

Note: If there is one set of brackets within another set, begin by calculating the inner-most brackets first. E.g. $7 + [2 \times (6 \times 5^2 + 3)]$.

Multiplication by Negative Numbers

When multiplying by negative numbers, the following rules apply depending on whether the other number is positive or negative.

A **negative** multiplied by a **positive** results in a negative. Eg. -3 x 7 = -21

A **negative** multiplied by a **negative** results in a positive. Eg. -3 x -7 = 21

Fractions

A fraction represents the division of the numerator by the denominator, and is written as,

$$\frac{\boldsymbol{x}}{\boldsymbol{y}}$$

Where $\boldsymbol{x}$ is the numerator, and $\boldsymbol{y}$ is the denominator.

For example, $\frac{1}{4}$ represents '1 divided by 4' and is equivalent to writing '1 ÷ 4'.

Simplifying fractions

Often fractions can be **simplified** into smaller numbers by dividing both the numerator and the denominator by a common factor.

For example,

$$\frac{5}{20} = \frac{(5 \div 5)}{(20 \div 5)} = \frac{1}{4}$$

So long as the numerator and denominator are divided by the same number, then the value of the fraction has not changed. This same principle applies to multiplying the numerator and denominator by the same number.

Multiplying fractions

Multiplication of fractions is the simplest manipulation of fractions, and involves multiplying the numerators together and the denominators together.

For example,

$$\frac{2}{7} \times \frac{5}{8}$$

$$= \frac{(2 \times 5)}{(7 \times 8)}$$

$$= \frac{10}{56} = \frac{5}{28}$$

When one fraction is multiplied by its reciprocal (i.e. an inversion of the other fraction), the result is 1. For example,

$$\frac{2}{7} \times \frac{7}{2}$$

$$= \frac{(7 \times 2)}{(2 \times 7)} = 1$$

Adding and subtracting fractions

When **adding** or **subtracting** fractions, there are two distinct steps: firstly, the fractions need to be manipulated such that they have a **common denominator**, and secondly, only the numerators are added together, while keeping the denominator the same.

To find the common denominator, consider what the smallest number is of which the two denominators are factors of. Alternatively, a common denominator will always be the product of the two denominators.

To manipulate the fractions so that they have a common denominator usually involves multiplying one, or both, of the fractions.

For example,

$$\frac{3}{5} + \frac{1}{3}$$

The common denominator is 15, and so you must multiply the first fraction by $\frac{3}{3}$, and the second fraction by $\frac{5}{5}$. By multiplying by $\frac{3}{3}$, it does not change the value of the fraction as you are just multiplying by 1.

$$\frac{3}{5} \times \frac{3}{3} + \frac{1}{3} \times \frac{5}{5}$$

$$= \frac{9}{15} + \frac{5}{15}$$

Now that the fractions have a common denominator, you can add the numerators together.

$$\frac{9}{15} + \frac{5}{15} = \frac{14}{15}$$

Dividing fractions

When dividing by a fraction, you must invert the fraction that you are dividing by and then perform multiplication.

For example,

$$\frac{3}{5} \div \frac{1}{3}$$

$$\frac{3}{5} \times \frac{3}{1}$$

$$= \frac{9}{5}$$

Decimals

All fractions can be expressed as decimals, and most decimals can be expressed as fractions. To express a decimal as a fraction, the number of zeros in the denominator is determined by the number of digits after the decimal point.

For example,

$$0.51$$

The numerator will be 51, and the denominator will have 2 zeros (i.e. it will be 100).

$$\frac{51}{100}$$

Multiplying Decimals by Powers of 10

When multiplying a decimal by a power of 10, the decimal is moved to the **right** by the number of the power. For example, if the decimal is multiplied by 10^2, then the decimal point is moved 2 places to the right.

For example,

$$0.5154 \times 10^2 = 51.54$$

Dividing Decimals by Powers of 10

When dividing a decimal by a power of 10, the decimal is moved to the **left** by the number of the power. This has the opposite effect of multiplying by a power of 10.

For example,

$$51.54 \div 10^2 = 0.5154$$

Percentages

A percent means 'out of a hundred', e.g. 25% means '25 out of every 100'. This is the same as the numerator of a fraction when the denominator is 100.

For example, 25% can be written as,

$$\frac{25}{100}$$

It is good practice to avoid mixing decimals with fractions. Therefore, 2.5% should not be written as $\frac{2.5}{100}$. Instead, both the numerator and denominator should be multiplied by multiples of 10 until the decimal disappears.

$$\frac{2.5}{100} \times \frac{10}{10} = \frac{25}{1000}$$

Calculating the percentage represented by a fraction

To calculate the percentage, multiply the quotient by 100. For example,

$$\frac{6}{60} = 0.10$$

$$0.10 \times 100 = 10\%$$

Multiplying a number by a percentage

When calculating a percentage (e.g. 25% of 24), you can convert the percentage into a fraction first, and then multiply.

For example,

$$\frac{25}{100} \times 24 = 6$$

Increasing a number by a percentage

When increasing a number by a given percentage, add that percentage to 100% and perform the multiplication procedure above.

For example, to increase $80 by 10%, multiply $80 by 110%.

$$\$80 \times 110\% = \$80 \times \frac{110}{100} = \$88$$

Alternatively, you can convert 110% to a decimal, such as 1.1.

Decreasing a number by a percentage

Just as you increase a number by a percentage, you can apply the same principle to decrease a number. To decrease by 10%, add that percentage to 100% and then **divide** the number by the new percentage.

For example, to decrease $88 by 10%, divide by 110%.

$$\$88 \div 110\% = \$80 \div \frac{110}{100} = \$72.73$$

Scientific notation

Scientific notation, or standard index form, is a way of representing long numbers so as to make it easier to read and also to minimise the risk of error. Using scientific notation, any number can be written as a smaller number multiplied by a multiple of ten.

For example,

- $2135 = 2.135 \times 10^3$
- $0.003498 = 3.498 \times 10^{-3}$