THE UNIVERSITY OF SYDNEY

Business School

Solving Linear and Quadratic Equations

Mathematics Help Sheet

The University of Sydney Business School

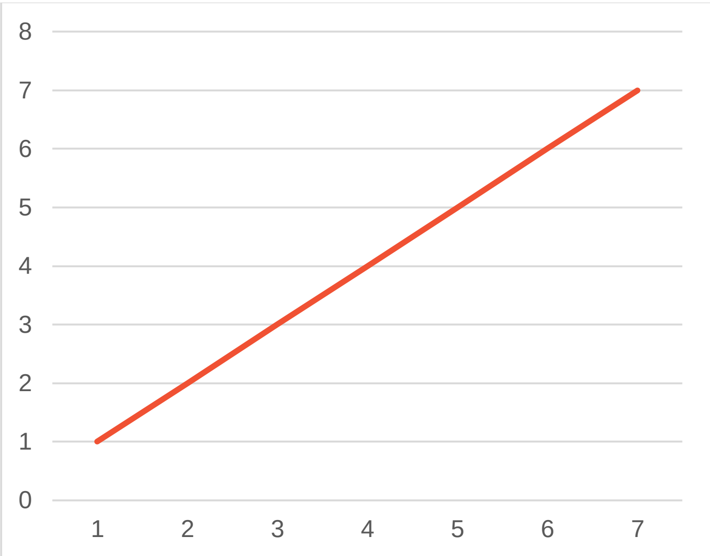
Introduction to Functions

Linear Functions

A linear function possesses the general form,

$$y = ax + b$$

Where x and y are variables and a and b are parameters. Hence, linear functions form a straight line when graphed on a Cartesian plane:



Dependent and Independent Variables

Consider the linear function,

$$y = ax + b$$

The variable on the left hand side, y, is called the **dependent variable** as its value depends on the value of the **independent variable**. The variable on the right hand side, x, is called the independent variable.

For example, given the two variables **"number of days of rain per month"** and **"number of umbrella sales"**, the number of days of rain would be the independent variable, whereas the number of umbrella sales would be the dependent variable. This is because the number of umbrella sales will likely depend on how much it has rained, rather than the other way around.

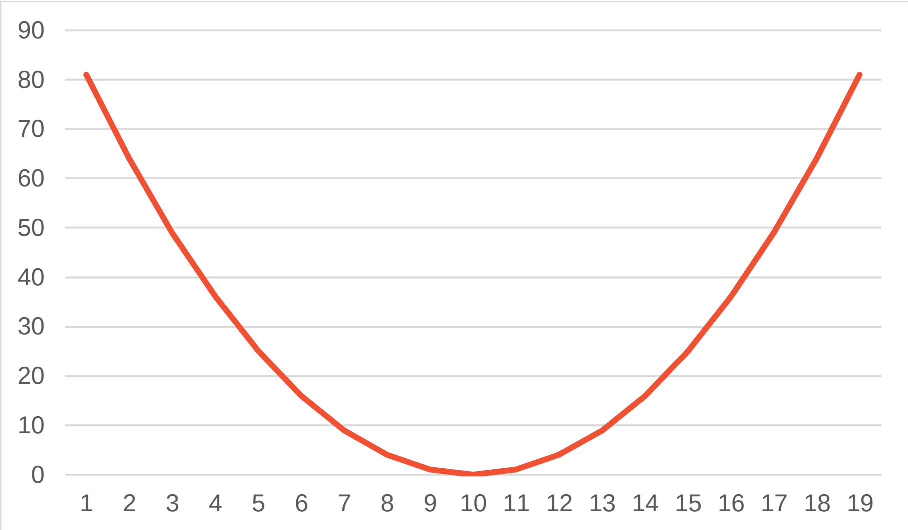
Quadratic Functions

A quadratic function possesses the general form,

$$y = ax^2 + bx + c$$

Where x and y are variables and a and b and c are parameters.

The important thing to note about quadratic functions is the presence of a squared variable term (e.g. x^2). Graphically, quadratic functions form a parabola:



When the coefficient of the squared term is **positive**, the parabola will be U-shaped like above. When the coefficient of the squared term is **negative**, the parabola will be upside-down.

One way to remember this more easily is that a positive coefficient results in a smiley face, whereas a negative coefficient results in a frowny face.

Solving Quadratic Equations

A quadratic equation can be factorised in order to find its **roots**. The roots of a quadratic equation are the values of x which make the equation equal to 0. They also represent the two places on the function that intersects the x-axis. Thus, “solving” a quadratic equation means finding its roots.

For example, consider the following quadratic function:

$$y = x^2 + 12x + 32$$

Step 1: Factorise

Recall that the above quadratic function can be factorised by finding the two values which add to 12 and multiply to get 32. Hence, the above function can be factorised into:

$$y = (x + 4)(x + 8)$$

Note: for a detailed discussion on factorisation, see the help sheet "Algebra"

Step 2: Find the Roots

Since multiplying anything by 0 results in 0, given the factorised form:

$$y = (x + 4)(x + 8)$$

The equation will equal 0 when any of the following two are true,

$$(x + 4) = 0$$
$$(x + 8) = 0$$

That is where, $x = -4$ or $x = -8$.

Hence, the roots of the equation are,

$$\boldsymbol{x = -4}$$
$$\boldsymbol{x = -8}$$

The Quadratic Equation

The quadratic equation can be used to find the roots of a quadratic function. It is particularly useful when the roots are not integers and hence factorisation will be difficult.

Given a function,

$$y = ax^2 + bx + c$$

The roots can be found by,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Where a, b and c are the coefficients of the quadratic function. The term under the square root, $b^2 - 4ac$, is called the **discriminant** and a function will only have real roots if the discriminant is positive i.e. $b^2 - 4ac \geq 0$

For example,

$$y = x^2 + 12x + 32$$

$$x = \frac{-12 \pm \sqrt{12^2 - 4(1)(32)}}{2(1)}$$

$$x = \frac{-12 \pm \sqrt{16}}{2}$$

$$x = 6 \pm 2$$

Hence, the roots of the equation are $x = 4$ and $x = 8$.

Simultaneous Equations

Simultaneous equations occur when there are two or more equations (regarding the same variables) that are true at the same time. Hence, solving a simultaneous equation will require you to **find the values** of each of the variables which make all equations hold true.

The Substitution Method

The substitution method involves substituting one equation into another, in order to solve the simultaneous equations.

For example,

$$P = 60 - Q \qquad (1)$$

$$P = Q + 30 \qquad (2)$$

Substituting the first equation into the second:

$$Q + 30 = 60 - Q$$

$$2Q = 30$$

$$\boldsymbol{Q = 15}$$

Now that we have found the first variable, we can substitute its value into either equation (1) or equation (2) in order to find the value of the second variable.

$$P = 60 - 15$$

$$\boldsymbol{P = 45}$$

The Elimination Method

The elimination method involves adding or subtracting one of the equations from another, in order to solve the simultaneous equations.

For example,

$$y = 4x \qquad (1)$$

$$2x + y = 12 \qquad (2)$$

The first step is to rearrange equation (2) to put y on the left hand side,

$$y = 12 - 2x \quad (3)$$

Now, subtract (1) from (3),

$$y - y = 12 - 2x - 4x$$

$$0 = 12 - 6x$$

$$6x = 12$$

$$\boldsymbol{x = 2}$$

Now that you have obtained the value for x, you can substitute $x = 2$ into either equation (1) or (2). Substituting into equation (1),

$$y = 4(2)$$

$$\boldsymbol{y = 8}$$

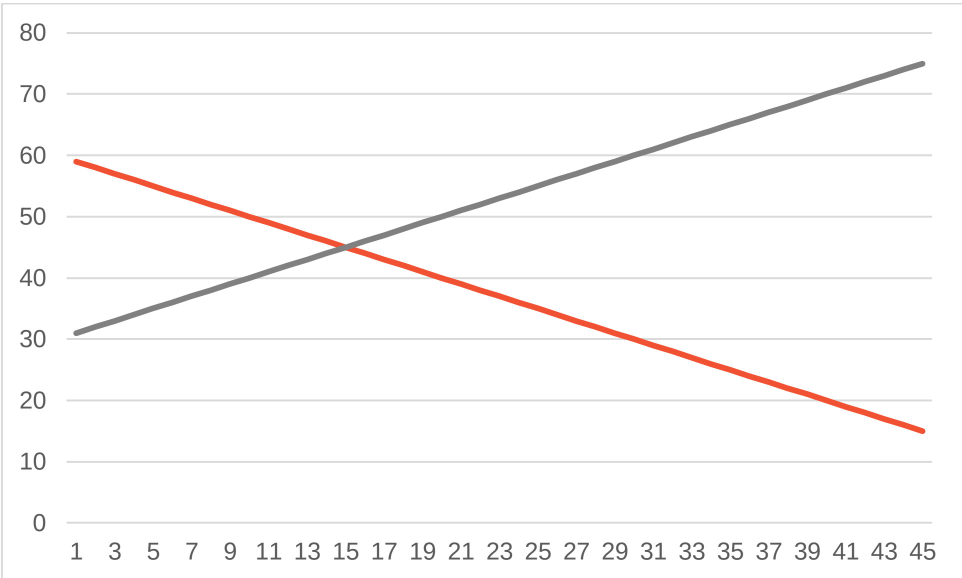
Solving Graphically

Graphically, the point of intersection of two simultaneous equations represents the co-ordinates which form the values that solve the simultaneous equations.

For example the following equations can be graphed on a Cartesian plane,

$$P = 60 - Q$$

$$P = Q + 30$$



The intersection point is at the coordinates (15, 45) which represent the values for (Q, P).

Solving Simultaneous Quadratic Equations

Solving quadratic equations simultaneously is more complicated algebraically but conceptually similar to solving linear simultaneous equations.

For example, consider the following simultaneous equations,

$$y = x^2 + x + 10 \quad (1)$$

$$y = 2x^2 + 4x + 5 \quad (2)$$

Substituting equation (1) into equation (2),

$$2x^2 + 4x + 5 = x^2 + x + 10$$

$$x^2 + 4x - x + 5 - 10 = 0$$

$$\boldsymbol{x^2 + 3x + -10 = 0} \qquad (\mathbf{3})$$

Factorising equation (3),

$$(x+5)(x-2) = 0$$

Thus, the roots of the equation are $x = -5$ and $x = 2$

Alternatively, you can apply the quadratic formula to (3),

$$x^2 + 3x + -10 = 0$$

$$x = \frac{-3 \pm \sqrt{3^2 - 4(1)(-10)}}{2(1)}$$

$$x = \frac{-3 \pm \sqrt{49}}{2}$$

$$x = \frac{-3 \pm 7}{2}$$

Hence, the roots of the equation are $x = -5$ and $x = 2$.