Addition, subtraction and scalar multiplication

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Addition of matrices

We can add matrices if they are the same size. If

$$A = \begin{bmatrix} 4 & -1 & 3 & 0 \\ 1 & -2 & 9 & -1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & -5 & 0 & -1 \\ -1 & -4 & 3 & -1 \end{bmatrix}$$

then we define $A + B$ as the matrix we get by adding the corresponding entries.

$$\begin{bmatrix} 4 & -1 & 3 & 0 \\ 1 & -2 & 9 & -1 \end{bmatrix} + \begin{bmatrix} 0 & -5 & 0 & -1 \\ -1 & -4 & 3 & -1 \end{bmatrix} = \begin{bmatrix} 4 & -6 & 3 & -1 \\ 0 & -6 & 12 & -2 \end{bmatrix}$$

Subtracting matrices

Similarly we define $A - B$ as the matrix we get when we subtract corresponding entries.

$$\begin{bmatrix} 4 & -1 & 3 & 0 \\ 1 & -2 & 9 & -1 \end{bmatrix} - \begin{bmatrix} 0 & -5 & 0 & -1 \\ -1 & -4 & 3 & -1 \end{bmatrix} = \begin{bmatrix} 4 & 4 & 3 & 1 \\ 2 & 2 & 6 & 0 \end{bmatrix}$$

Note that we cannot add or subtract matrices that are different sizes.

$$\begin{bmatrix} 4 & -1 & 3 & 0 \\ 1 & -2 & 9 & -1 \end{bmatrix} - \begin{bmatrix} 0 & -5 & 0 \\ -1 & -4 & 3 \\ 6 & -2 & 1 \end{bmatrix} \text{ does not make sense.}$$

Scalar multiplication of matrices

We can multiply a matrix by a scalar.

$$\text{Let } A = \begin{bmatrix} 0 & -5 & 0 \\ -1 & -4 & 3 \\ 6 & -2 & 1 \end{bmatrix} \text{ then } 3A = \begin{bmatrix} 0 & -15 & 0 \\ -3 & -12 & 9 \\ 18 & -6 & 3 \end{bmatrix}.$$

We have multiplied each entry in A by the scalar 3.

$$\text{Let } B = \begin{bmatrix} -15 & 0 \\ -3 & -12 \\ -6 & 3 \end{bmatrix} \text{ then } -2B = \begin{bmatrix} 30 & 0 \\ 6 & 24 \\ 12 & -6 \end{bmatrix}.$$

Some simple properties of matrices

Let A, B and C be matrices that are the same size, then

$$A + B = B + A, \qquad A + (B + C) = (A + B) + C.$$

Where 0 is the zero matrix that is the same size as A,

$$A + 0 = 0 + A = A, \qquad A + (-1)A = A - A = 0.$$

If α and β are scalars then

$$\alpha(\beta A) = (\alpha\beta)A, \quad \alpha(A + B) = \alpha A + \alpha B, \quad (\alpha + \beta)A = \alpha A + \beta A.$$

We will demonstrated one of these in the next slide but will leave the rest for you.

Demonstrating one of the properties

Let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$, $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$, and $C = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}$.

Then

$$\begin{aligned} A + (B + C) &= \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} + \begin{bmatrix} b_{11} + c_{11} & b_{12} + c_{12} \\ b_{21} + c_{21} & b_{22} + c_{22} \end{bmatrix} \\ &= \begin{bmatrix} a_{11} + b_{11} + c_{11} & a_{12} + b_{12} + c_{12} \\ a_{21} + b_{21} + c_{21} & a_{22} + b_{22} + c_{22} \end{bmatrix} \\ &= \begin{bmatrix} a_{11} + b_{11} & a_{12} + b_{12} \\ a_{21} + b_{21} & a_{22} + b_{22} \end{bmatrix} + \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} \\ &= (A + B) + C. \end{aligned}$$