An Introduction to Matrix Algebra

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An introduction to matrices

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What is a matrix?

A matrix is an array of numbers.

$$\begin{bmatrix} 4 & -1 & 3 & 0 \\ 1 & -2 & 9 & -1 \end{bmatrix}$$

The size of the matrix is determined by its number of rows and number of columns.

The matrix above is a 2 by 4 matrix. That is, it has 2 rows and 4 columns. We write this as 2×4.

Row and column matrices

A matrix with only one row is called a row matrix or row vector.

$$\begin{bmatrix} 4 & -1 & 3 & 0 \end{bmatrix}$$

A matrix with only one column is called a column matrix or column vector.

$$\begin{bmatrix} 4 \\ -1 \\ 3 \\ 0 \end{bmatrix}$$

Square matrices and zero matrices

A matrix with the same number of rows and columns is called a square matrix.

$$\begin{bmatrix} 4 & -1 & 3 \\ -1 & 0 & -1 \\ 1 & 3 & -2 \end{bmatrix}$$ is a 3×3 square matrix.

If we have a matrix where every entry is zero, this matrix is called a zero matrix.

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$ is the 3×4 zero matrix.

Identity matrices

A square matrix which has 1's on the diagonal and 0's everywhere else is called an *identity* matrix.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ is the 3×3 identity matrix.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ is the 4×4 identity matrix.

Labelling entries of a matrix

Consider the following matrix which we shall call A.

$$A = \begin{bmatrix} 1 & 0 & -1 & 6 \\ 7 & 1 & 0 & -2 \\ 0 & 3 & 1 & 0 \end{bmatrix} \qquad \text{is a } 3 \times 4 \text{ matrix.}$$

The $(i, j)th$ entry of A is the entry in the ith row and the jth column of A.

For example, the $(3, 2)th$ entry of A is 3.

We often refer to the $(i, j)th$ entry of A as a_{ij}.

The transpose of a matrix

Consider the following matrix $A = \begin{bmatrix} 1 & 0 & -1 & 6 \\ 7 & 1 & 0 & -2 \\ 0 & 3 & 1 & 0 \end{bmatrix}$.

We define the transpose of A, A', as the matrix whose $(i,j)th$ entry is the $(j,i)th$ entry of A.

The $(2,3)th$ entry of A' is the $(3,2)th$ entry of A, ie 3.
So,

$$A' = \begin{bmatrix} 1 & 7 & 0 \\ 0 & 1 & 3 \\ -1 & 0 & 1 \\ 6 & -2 & 0 \end{bmatrix}.$$

Notice that while A is a 3×4 matrix, A' is a 4×3 matrix, and the rows of A are the columns of A'.