

2 Unit Bridging Course - Day 3

Quadratic Equations

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A quadratic equation has a standard form of:

$$ax^2 + bx + c = 0$$

where a , b , c are constants $a \neq 0$.

If we can factorise the expression $ax^2 + bx + c$, we do so.

As the product of the factors are equal to zero, then one or other of the factors must be equal to zero. We then solve the resulting equations for x .

For example, solve $x^2 + 2x - 3 = 0$.

$$\begin{aligned}x^2 + 2x - 3 &= 0 \\(x + 3)(x - 1) &= 0\end{aligned}$$

Either $(x + 3) = 0$ or $(x - 1) = 0$. Hence $x = -3$ or $x = 1$.
Substitute to check:

$$x^2 + 2x - 3 = (-3)^2 + 2 \times -3 - 3 = 9 - 6 - 3 = 0$$

$$x^2 + 2x - 3 = 1^2 + 2 \times 1 - 3 = 1 + 2 - 3 = 0$$

Example

Solve $x^2 + 4x + 9 = 5$

First we rewrite the equation as

$$x^2 + 4x + 4 = 0$$

then factorise to give

$$(x + 2)(x + 2) = 0$$

so $x = -2$ is the solution.

Note that this quadratic equation only has 1 solution.

Example

Solve $4x^2 - 16 = 0$.

$$4x^2 - 16 = 0$$

$$(2x - 4)(2x + 4) = 0$$

so either $2x - 4 = 0$ or $2x + 4 = 0$.

Therefore the solution is $x = 2$ or $x = -2$.

Some quadratics are difficult or cannot be factorised.
If a solution exists, we can solve it using the quadratic formula.

Quadratic Formula

For $ax^2 + bx + c = 0$ where a , b and c are the constants and $a \neq 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Use the formula by simply plugging in the values from the quadratic equation.

Note that if $b^2 - 4ac < 0$, then the equation has no real solutions.

Example

Solve $2x^2 + 4x - 2 = 0$.

For this equation $a = 2$, $b = 4$, $c = -2$.

Plug them into the formula.

$$\begin{aligned}x &= \frac{-4 \pm \sqrt{4^2 - 4(2)(-2)}}{2(2)} = \frac{-4 \pm \sqrt{32}}{4} \\ &= \frac{-4 \pm \sqrt{16}\sqrt{2}}{4} = -1 \pm \sqrt{2}\end{aligned}$$

Therefore $x = -1 + \sqrt{2}$ or $x = -1 - \sqrt{2}$.

Example

Solve $4x^2 + 2x + 5 = 0$.

For this equation $a = 4$, $b = 2$, $c = 5$.

Notice that $b^2 - 4ac = 2^2 - 4(4)(5) = -76$, which is a negative number.

Therefore $4x^2 + 2x + 5 = 0$ has no real solutions.

Practice Questions

Solve the following.

1. $x^2 + 4x + 3 = 0$

2. $x^2 - 9 = 0$

3. $x^2 - 6x - 4 = -13$

4. $3x^2 + 2x - 1 = 0$

5. $4m^2 + 5m + 1 = 0$

6. $9x^2 - 2x - 5 = -1 - 2x$

7. $-2n^2 + 2x + 1 = 0$

8. $2x^2 - 3 = 0$

9. $4x^2 + x + 2 = 0$

10. $3x^2 - 5x = 0$

Answers to the practice questions.

1. $x = -3$ or $x = -1$

2. $x = 3$ or $x = -3$

3. $x = 3$

4. $x = \frac{1}{3}$ or $x = -1$

5. $m = -\frac{1}{4}$ or $m = -1$

6. $x = \frac{2}{3}$ or $x = -\frac{2}{3}$

7. $n = \frac{1+\sqrt{3}}{2}$ or $n = \frac{1-\sqrt{3}}{2}$

8. $x = \frac{\sqrt{6}}{2}$ or $x = -\frac{\sqrt{6}}{2}$

9. No real solutions.

10. $x = 0$ or $x = \frac{5}{3}$